COMPACT-MDD: EFFICIENTLY FILTERING (S)MDD CONSTRAINTS WITH REVERSIBLE SPARSE BIT-SET

IJCAI18

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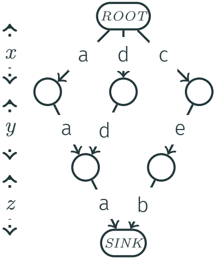
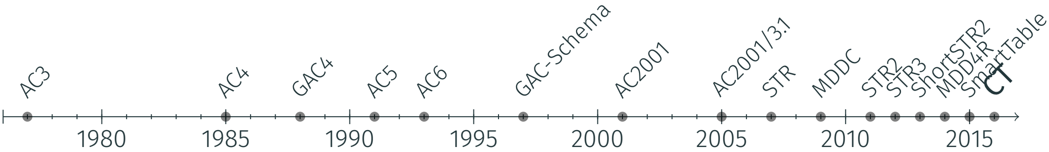
THE EXTENSIONAL CONSTRAINT: TABLES AND MDDS

	x	y	z
τ_1	a	a	a
τ_2	d	d	a
τ_3	c	e	b
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Tables are the oldest most used CP constraints

MDDs are equivalent to tables





2016 : New algorithm! Compact-Table [CP2016], based on bitwise operations, completely outperformed existing algorithms

THE MDD AND SMDD DATA STRUCTURES

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

MDD

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
	1	0	1	0
	1	0	1	1
	1	1	0	1
1	0	0	0	1
	1	0	0	0
	1	1	0	0
	1	1	0	1

MDD

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

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<tr><td rowspan="3">1</td><td>0</td><td>1</td><td>0</td></tr>
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x_1	x_2	x_3	x_4	x_5
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0	1	0	1	0
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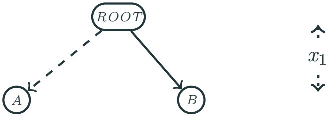

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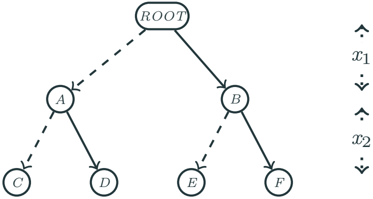

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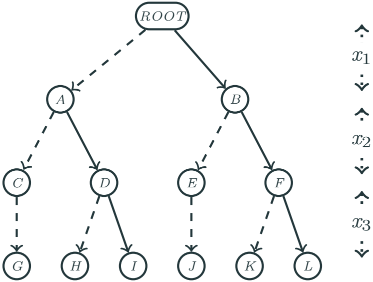

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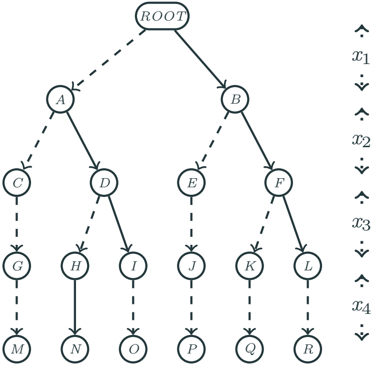

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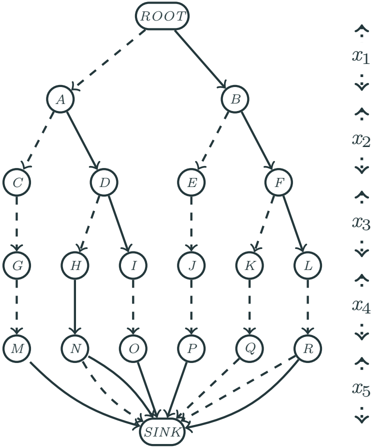

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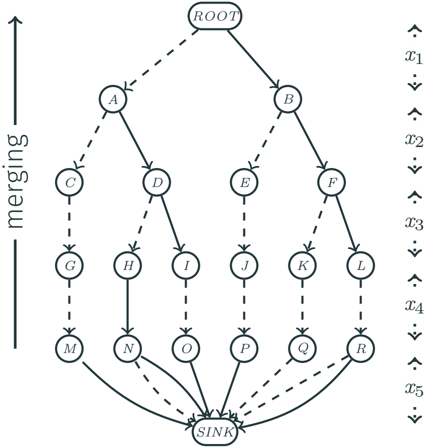

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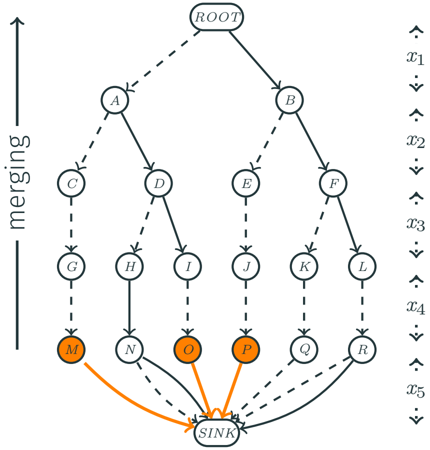

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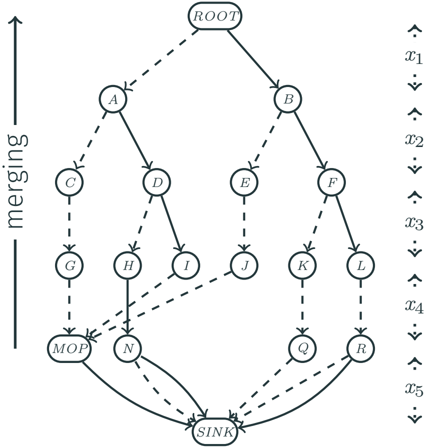

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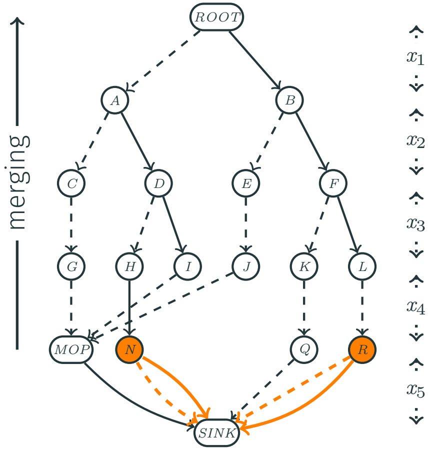

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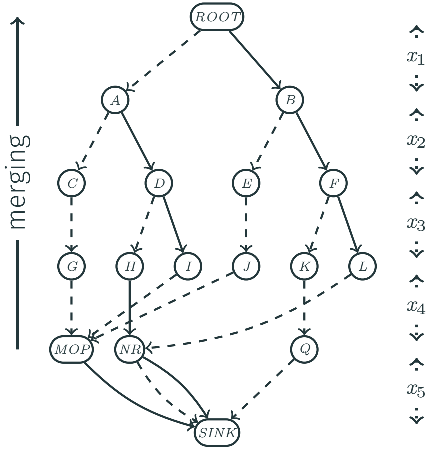

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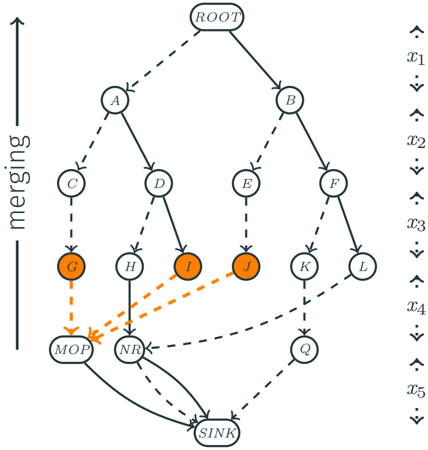
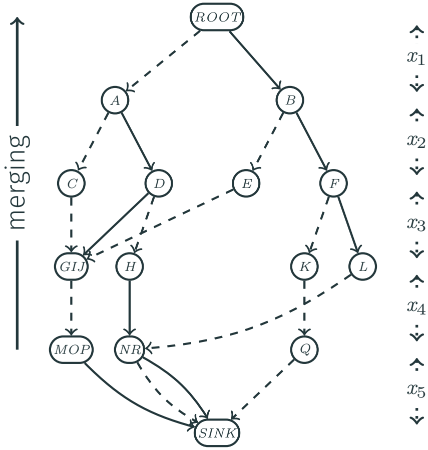


MDD

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

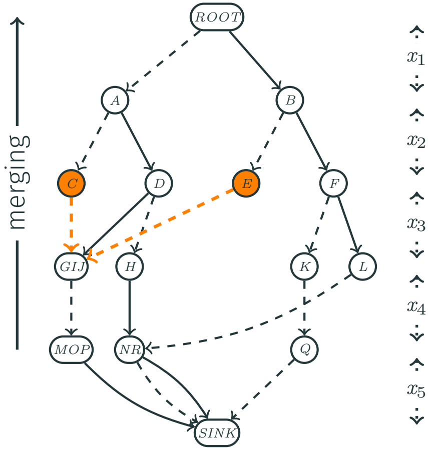
<table>
<tr><th>x_1</th><th>x_2</th><th>x_3</th><th>x_4</th><th>x_5</th></tr>
<tr><td rowspan="4">0</td><td>0</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td rowspan="3">1</td><td rowspan="2">0</td><td rowspan="2">1</td><td>0</td></tr>
<tr><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td></tr>
<tr><td rowspan="4">1</td><td>0</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td rowspan="3">1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td rowspan="2">1</td><td rowspan="2">0</td><td>0</td></tr>
<tr><td>1</td></tr>
</table>



MDD

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

<table>
<tr><th>x_1</th><th>x_2</th><th>x_3</th><th>x_4</th><th>x_5</th></tr>
<tr><td rowspan="4">0</td><td>0</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td rowspan="3">1</td><td rowspan="2">0</td><td rowspan="2">1</td><td>0</td></tr>
<tr><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td></tr>
<tr><td rowspan="4">1</td><td>0</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td rowspan="3">1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td rowspan="2">1</td><td rowspan="2">0</td><td>0</td></tr>
<tr><td>1</td></tr>
</table>



x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

<table>
<tr><th>x_1</th><th>x_2</th><th>x_3</th><th>x_4</th><th>x_5</th></tr>
<tr><td rowspan="4">0</td><td>0</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td rowspan="3">1</td><td rowspan="2">0</td><td rowspan="2">1</td><td>0</td></tr>
<tr><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td></tr>
<tr><td rowspan="4">1</td><td>0</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td rowspan="3">1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td rowspan="2">1</td><td rowspan="2">0</td><td>0</td></tr>
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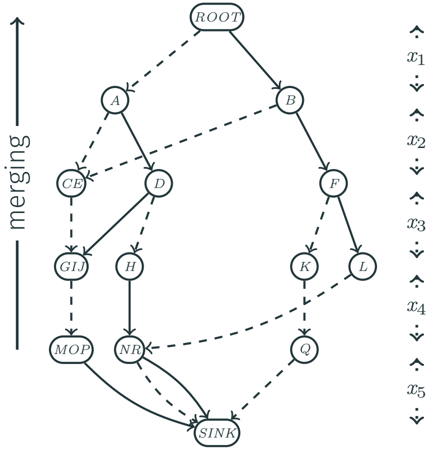
</table>



x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

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x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
0	1
1	0
1	1

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
0	1
1	0
1	1

suffixes

x_4	x_5
0	0
1	0
0	1
1	1

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	0
0	1
1	1

SMDD

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	
0	1
1	

x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	
0	1
1	

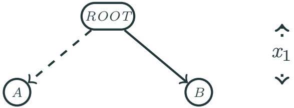
x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	
0	1
1	



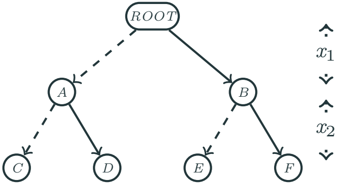
x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	
0	1
1	



x_1	x_2	x_3	x_4	x_5
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0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

<table>
<tr><th>x_1</th><th>x_2</th></tr>
<tr><td rowspan="2">0</td><td>0</td></tr>
<tr><td>1</td></tr>
<tr><td rowspan="2">1</td><td>0</td></tr>
<tr><td>1</td></tr>
</table>

suffixes

<table>
<tr><th>x_4</th><th>x_5</th></tr>
<tr><td>0</td><td rowspan="2">0</td></tr>
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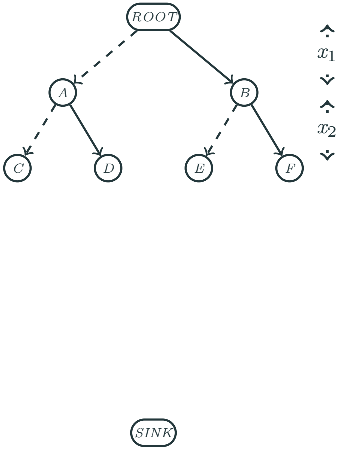

x_1	x_2	x_3	x_4	x_5
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0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

<table>
<tr><th>x_1</th><th>x_2</th></tr>
<tr><td rowspan="2">0</td><td>0</td></tr>
<tr><td>1</td></tr>
<tr><td rowspan="2">1</td><td>0</td></tr>
<tr><td>1</td></tr>
</table>

suffixes

<table>
<tr><th>x_4</th><th>x_5</th></tr>
<tr><td>0</td><td rowspan="2">0</td></tr>
<tr><td>1</td></tr>
<tr><td>0</td><td rowspan="2">1</td></tr>
<tr><td>1</td></tr>
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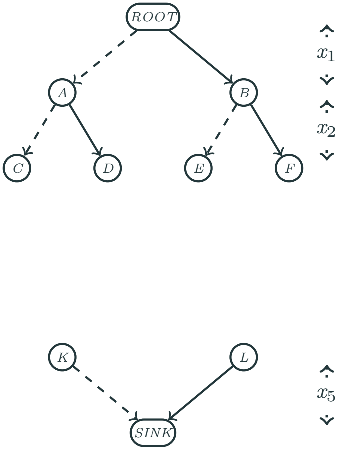

x_1	x_2	x_3	x_4	x_5
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0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

<table>
<tr><th>x_1</th><th>x_2</th></tr>
<tr><td rowspan="2">0</td><td>0</td></tr>
<tr><td>1</td></tr>
<tr><td rowspan="2">1</td><td>0</td></tr>
<tr><td>1</td></tr>
</table>

suffixes

<table>
<tr><th>x_4</th><th>x_5</th></tr>
<tr><td>0</td><td rowspan="2">0</td></tr>
<tr><td>1</td></tr>
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<tr><td>1</td></tr>
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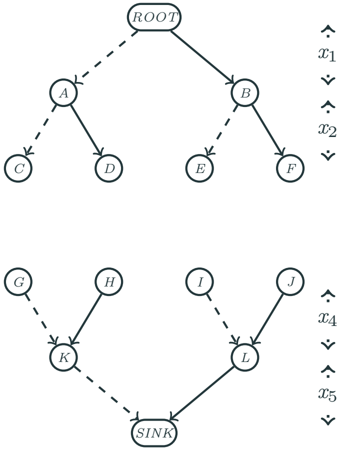


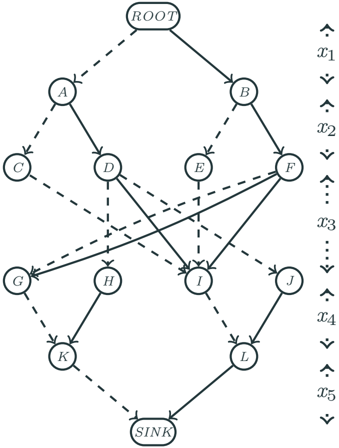
x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	
0	1
1	



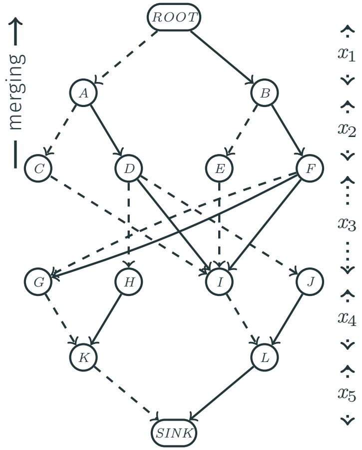
x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

<table>
<tr><th>x_1</th><th>x_2</th></tr>
<tr><td rowspan="2">0</td><td>0</td></tr>
<tr><td>1</td></tr>
<tr><td rowspan="2">1</td><td>0</td></tr>
<tr><td>1</td></tr>
</table>

suffixes

<table>
<tr><th>x_4</th><th>x_5</th></tr>
<tr><td>0</td><td rowspan="2">0</td></tr>
<tr><td>1</td></tr>
<tr><td>0</td><td rowspan="2">1</td></tr>
<tr><td>1</td></tr>
</table>



x_1	x_2	x_3	x_4	x_5
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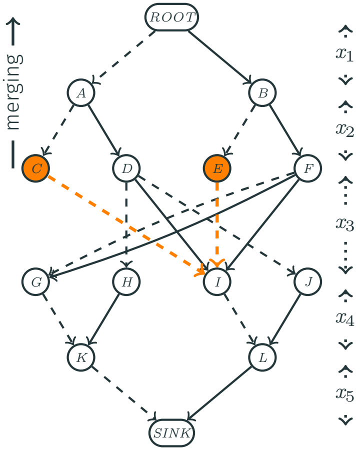
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	
0	1
1	



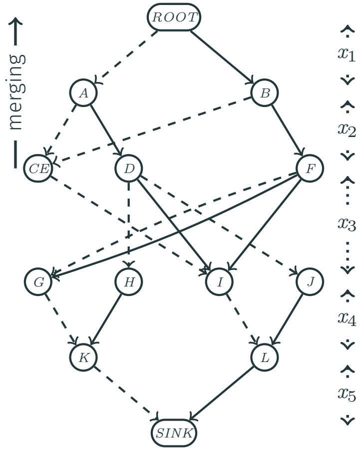
x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	
0	1
1	



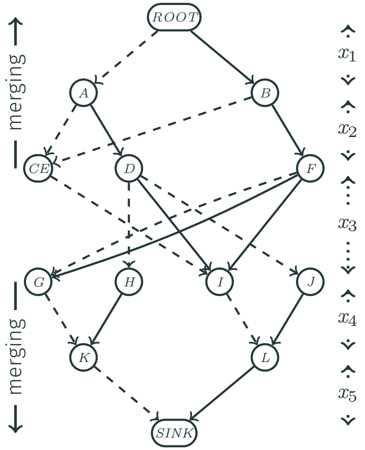
x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	
0	1
1	



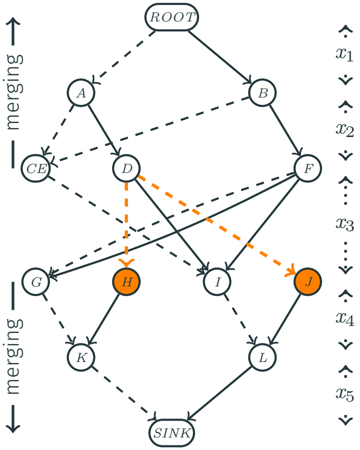
x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

prefixes

x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	
0	1
1	



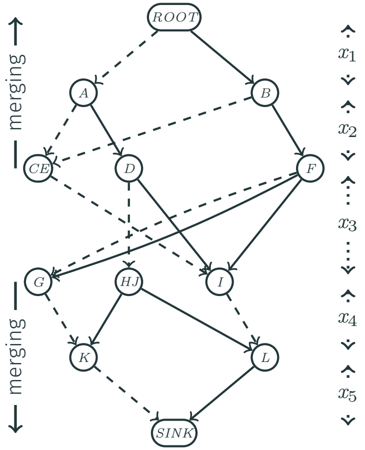
x_1	x_2	x_3	x_4	x_5
0	0	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	0
1	1	1	0	0
1	1	1	0	1

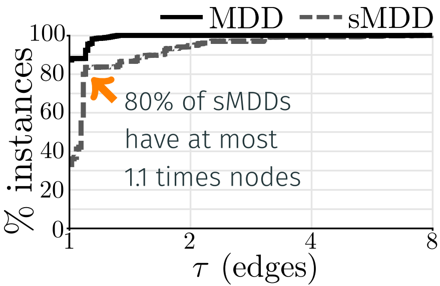
prefixes

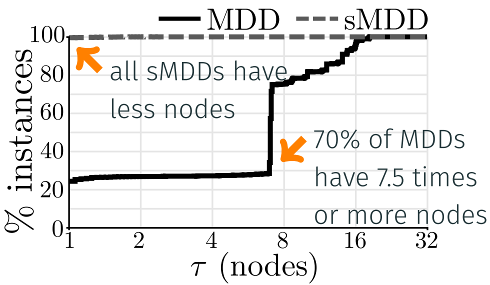
x_1	x_2
0	0
	1
1	0
	1

suffixes

x_4	x_5
0	0
1	
0	1
1	

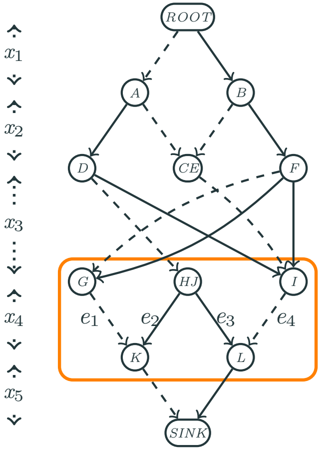






sMDDs: **Less few** nodes but **a bit more** edges than MDDs

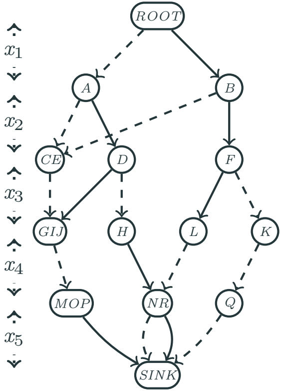
THE COMPACT-MDD ALGORITHM



Name	Set	Bit-set
`currArcs`$[x_4]$	$\{e_1, e_2, e_3, e_4\}$	[1 1 1 1]
`supports`$[x_4, 0]$	$\{e_1, \cancel{e_2}, \cancel{e_3}, e_4\}$	[1 0 0 1]
`arcsT`$[HJ, x_4]$	$\{\cancel{e_1}, e_2, e_3, \cancel{e_4}\}$	[0 1 0 0]
`arcsH`$[x_4, K]$	$\{e_1, e_2, \cancel{e_3}, \cancel{e_4}\}$	[1 1 0 0]

CMDD: UPDATE

COMPACT-MDD: UPDATE



MDD

currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 1 1 1 1]

currArcs[x_4]
[1 1 1 1]

currArcs[x_5]
[1 1 1 1]

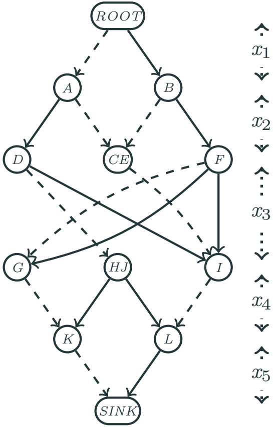
currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 1 1 1 1 1]

currArcs[x_4]
[1 1 1 1]

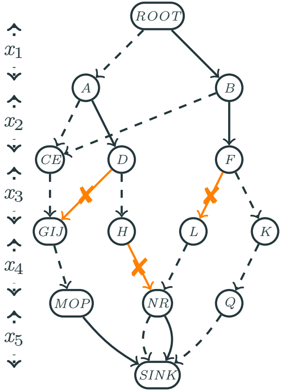
currArcs[x_5]
[1 1]



sMDD

COMPACT-MDD: UPDATE

Direct removal



currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 1 1 1 1]

currArcs[x_4]
[1 1 1 1]

currArcs[x_5]
[1 1 1 1]

MDD

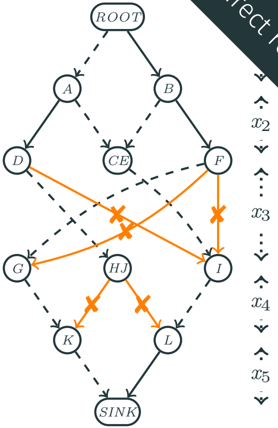
currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 1 1 1 1 1]

currArcs[x_4]
[1 1 1 1]

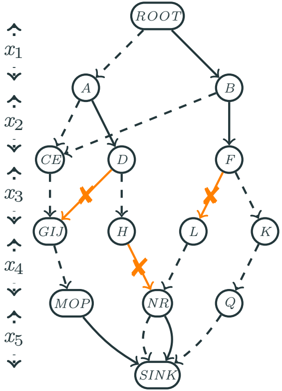
currArcs[x_5]
[1 1]



sMDD

COMPACT-MDD: UPDATE

Direct removal



MDD

currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 0 1]

currArcs[x_4]
[1 0 1 1]

currArcs[x_5]
[1 1 1 1]

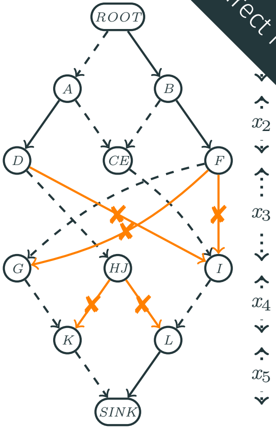
currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 1 0 0]

currArcs[x_4]
[1 0 0 1]

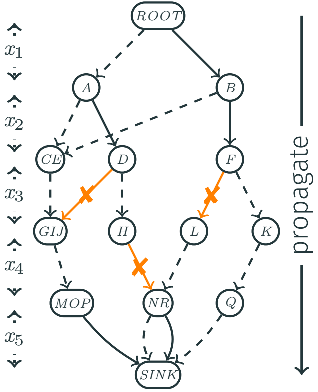
currArcs[x_5]
[1 1]



sMDD

COMPACT-MDD: UPDATE

Top down



currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 0 1]

currArcs[x_4]
[1 0 1 1]

currArcs[x_5]
[1 1 1 1]

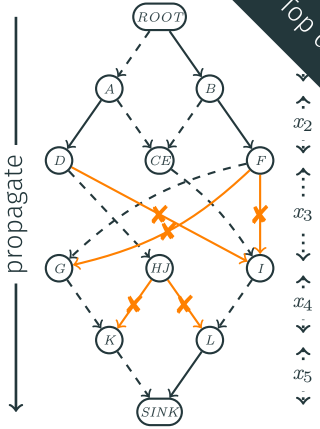
currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 1 0 0]

currArcs[x_4]
[1 0 0 1]

currArcs[x_5]
[1 1]

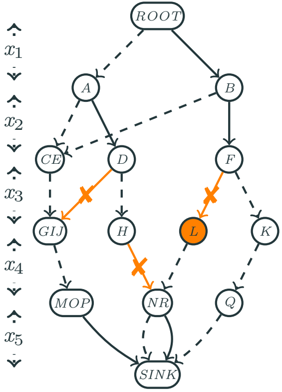


MDD

sMDD

COMPACT-MDD: UPDATE

Top down



MDD

propagate

currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 0 1]

currArcs[x_4]
[1 0 1 1]

currArcs[x_5]
[1 1 1 1]

currArcs[x_1]
[1 1]

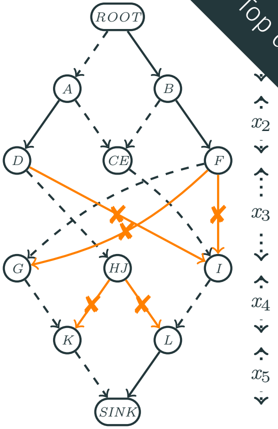
currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 1 0 0]

currArcs[x_4]
[1 0 0 1]

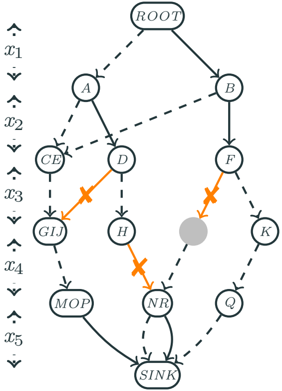
currArcs[x_5]
[1 1]

propagate



sMDD

Top down



propagate

currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 0 1]

currArcs[x_4]
[1 0 1 1]

currArcs[x_5]
[1 1 1 1]

MDD

currArcs[x_1]
[1 1]

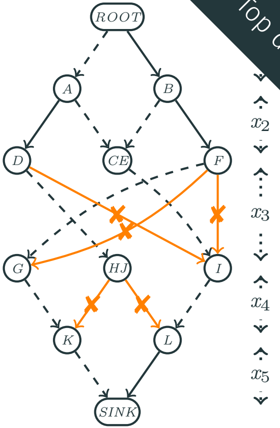
currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 1 0 0]

currArcs[x_4]
[1 0 0 1]

currArcs[x_5]
[1 1]

propagate



sMDD

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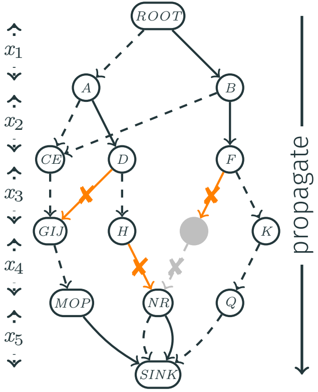
Top down



currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 0 1]

currArcs[x_4]
[1 0 0 1]

currArcs[x_5]
[1 1 1 1]

MDD

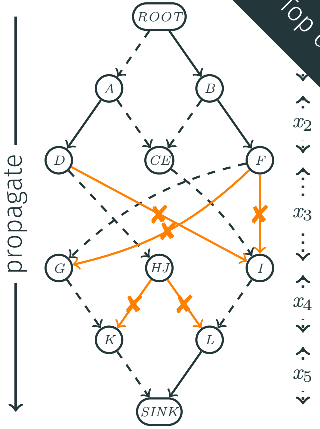
currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 1 0 0]

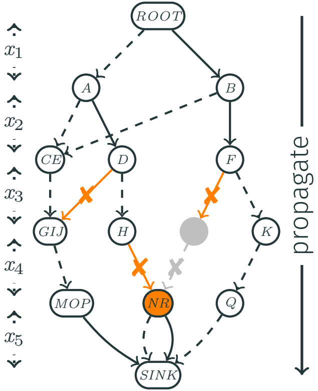
currArcs[x_4]
[1 0 0 1]

currArcs[x_5]
[1 1]



sMDD

Top down



MDD

currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 0 1]

currArcs[x_4]
[1 0 0 1]

currArcs[x_5]
[1 1 1 1]

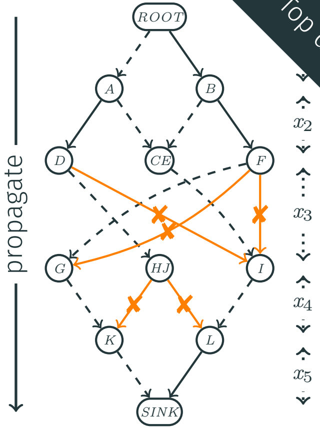
currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 1 0 0]

currArcs[x_4]
[1 0 0 1]

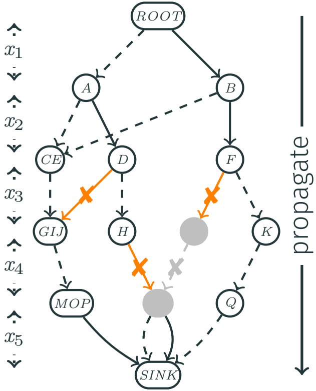
currArcs[x_5]
[1 1]



sMDD

COMPACT-MDD: UPDATE

Top down



MDD

currArcs[x_1] [1 1]

currArcs[x_2] [1 1 1 1]

currArcs[x_3] [1 0 1 0 1]

currArcs[x_4] [1 0 0 1]

currArcs[x_5] [1 1 1 1]

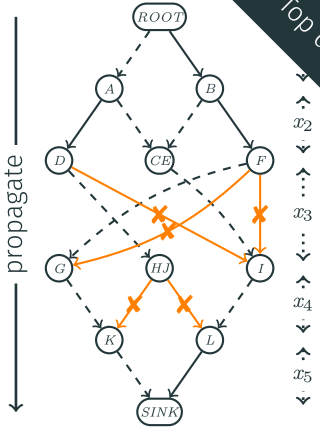
currArcs[x_1] [1 1]

currArcs[x_2] [1 1 1 1]

currArcs[x_3] [1 0 1 1 0 0]

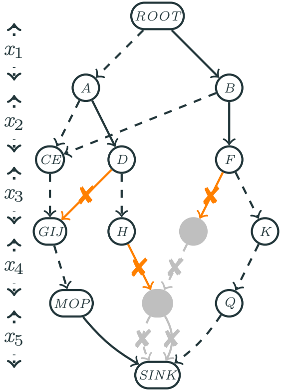
currArcs[x_4] [1 0 0 1]

currArcs[x_5] [1 1]



sMDD

Top down



propagate

$\texttt{currArcs}[x_1]$ [1 1]

$\texttt{currArcs}[x_2]$ [1 1 1 1]

$\texttt{currArcs}[x_3]$ [1 0 1 0 1]

$\texttt{currArcs}[x_4]$ [1 0 0 1]

$\texttt{currArcs}[x_5]$ [1 0 0 1]

MDD

$\texttt{currArcs}[x_1]$ [1 1]

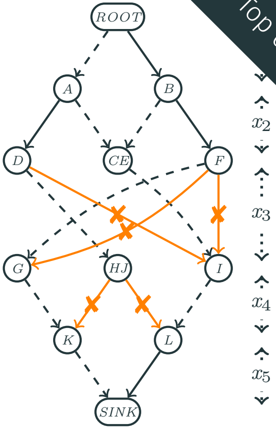
$\texttt{currArcs}[x_2]$ [1 1 1 1]

$\texttt{currArcs}[x_3]$ [1 0 1 1 0 0]

$\texttt{currArcs}[x_4]$ [1 0 0 1]

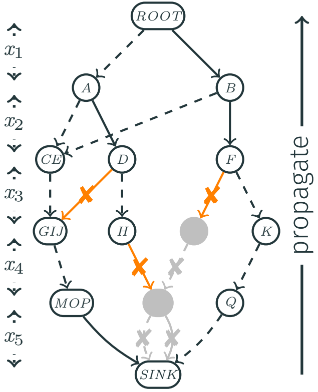
$\texttt{currArcs}[x_5]$ [1 1]

propagate



sMDD

Bottom up



currArcs[x_1] [1 1]

currArcs[x_2] [1 1 1 1]

currArcs[x_3] [1 0 1 0 1]

currArcs[x_4] [1 0 0 1]

currArcs[x_5] [1 0 0 1]

MDD

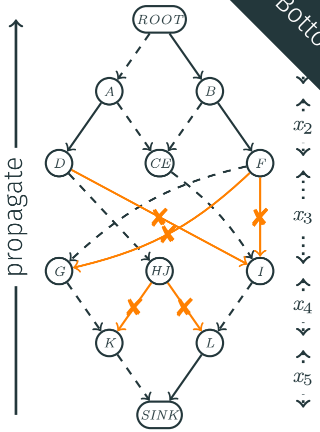
currArcs[x_1] [1 1]

currArcs[x_2] [1 1 1 1]

currArcs[x_3] [1 0 1 1 0 0]

currArcs[x_4] [1 0 0 1]

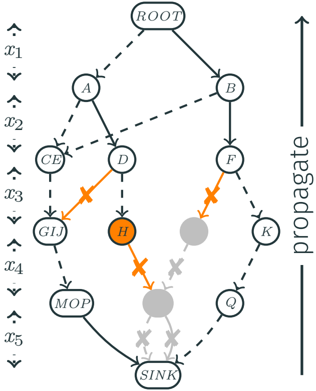
currArcs[x_5] [1 1]



sMDD

COMPACT-MDD: UPDATE

Bottom up



currArcs$[x_1]$ [1 1]

currArcs$[x_2]$ [1 1 1 1]

currArcs$[x_3]$ [1 0 1 0 1]

currArcs$[x_4]$ [1 0 0 1]

currArcs$[x_5]$ [1 0 0 1]

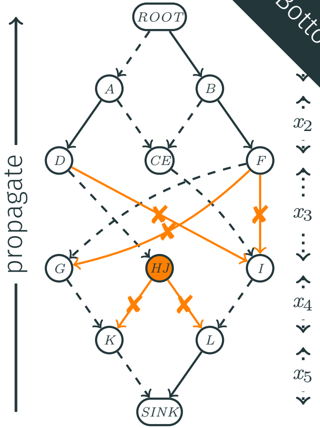
currArcs$[x_1]$ [1 1]

currArcs$[x_2]$ [1 1 1 1]

currArcs$[x_3]$ [1 0 1 1 0 0]

currArcs$[x_4]$ [1 0 0 1]

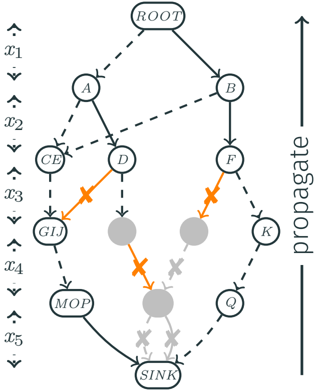
currArcs$[x_5]$ [1 1]



MDD

sMDD

COMPACT-MDD: UPDATE



MDD

currArcs$[x_1]$ [1 1]

currArcs$[x_2]$ [1 1 1 1]

currArcs$[x_3]$ [1 0 1 0 1]

currArcs$[x_4]$ [1 0 0 1]

currArcs$[x_5]$ [1 0 0 1]

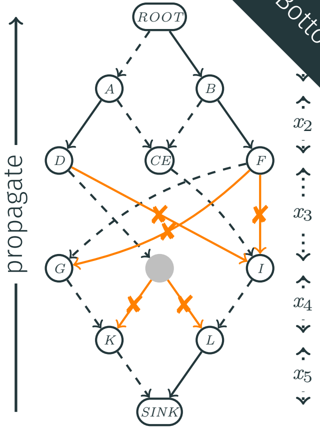
currArcs$[x_1]$ [1 1]

currArcs$[x_2]$ [1 1 1 1]

currArcs$[x_3]$ [1 0 1 1 0 0]

currArcs$[x_4]$ [1 0 0 1]

currArcs$[x_5]$ [1 1]

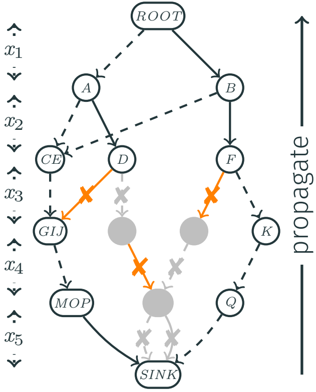


sMDD

Bottom up

COMPACT-MDD: UPDATE

Bottom up



currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 0 0 1]

currArcs[x_4]
[1 0 0 1]

currArcs[x_5]
[1 0 0 1]

MDD

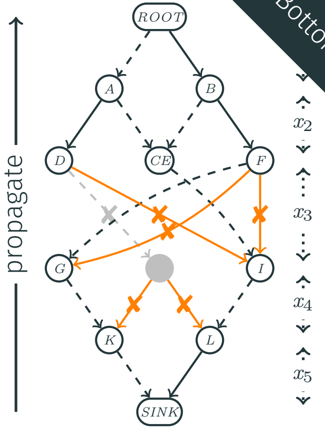
currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[0 0 1 1 0 0]

currArcs[x_4]
[1 0 0 1]

currArcs[x_5]
[1 1]



sMDD

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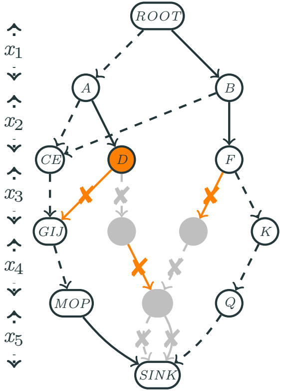
Bottom up



propagate

currArcs[x_1] [1 1]

currArcs[x_2] [1 1 1 1]

currArcs[x_3] [1 0 0 0 1]

currArcs[x_4] [1 0 0 1]

currArcs[x_5] [1 0 0 1]

MDD

currArcs[x_1] [1 1]

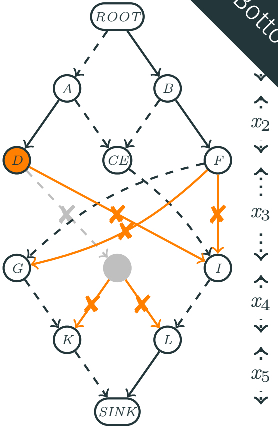
currArcs[x_2] [1 1 1 1]

currArcs[x_3] [0 0 1 1 0 0]

currArcs[x_4] [1 0 0 1]

currArcs[x_5] [1 1]

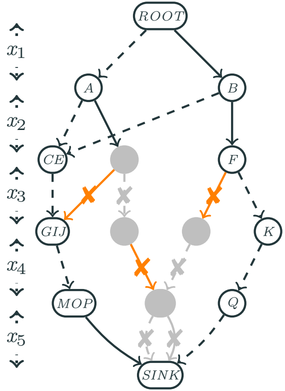
propagate



sMDD

COMPACT-MDD: UPDATE

Bottom up



propagate

currArcs[x_1] [1 1]

currArcs[x_2] [1 1 1 1]

currArcs[x_3] [1 0 0 0 1]

currArcs[x_4] [1 0 0 1]

currArcs[x_5] [1 0 0 1]

MDD

currArcs[x_1] [1 1]

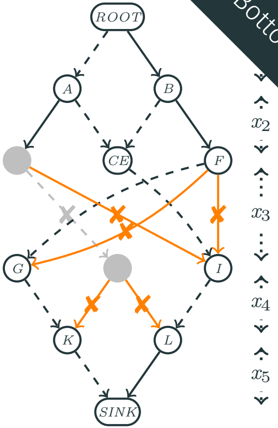
currArcs[x_2] [1 1 1 1]

currArcs[x_3] [0 0 1 1 0 0]

currArcs[x_4] [1 0 0 1]

currArcs[x_5] [1 1]

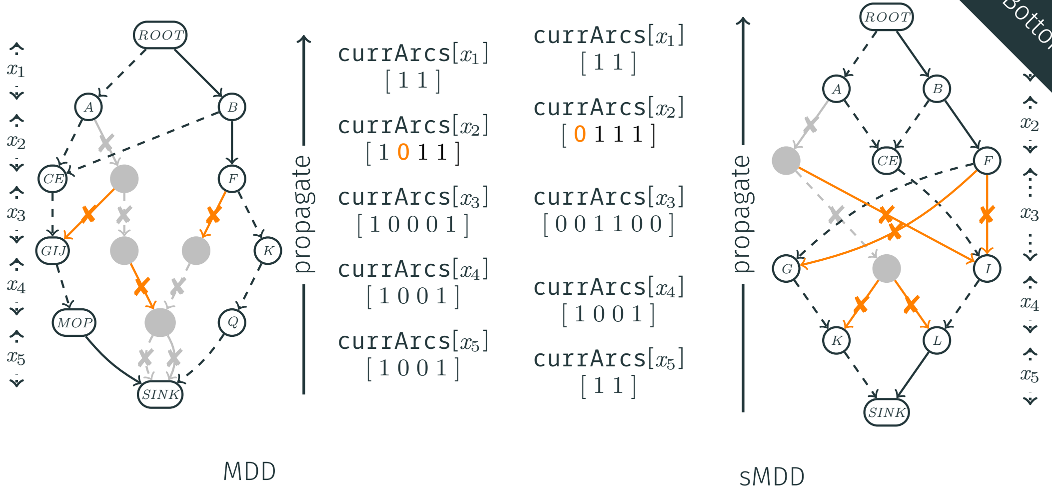
propagate



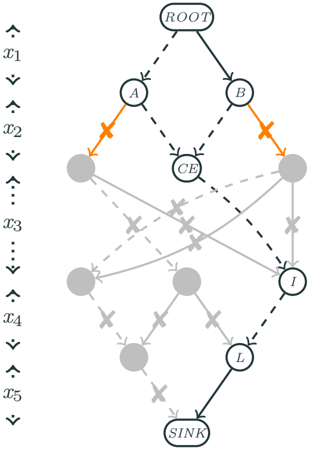
sMDD

COMPACT-MDD: UPDATE

Bottom up



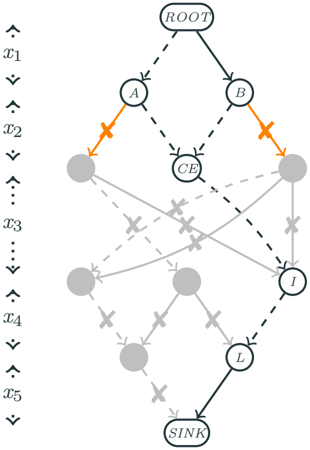
COMPACT-MDD: FILTERING



$\Delta_{x_2} = \{1\}$

x_1	x_2	x_3	x_4	x_5
$\{0,1\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$

(x,v)	currArcs[x]	supports[x,v]	$\cap$
$(x_1,0)$	11	10	10
$(x_1,1)$	11	01	01
$(x_3,0)$	001000	101100	001000
$(x_3,1)$	001000	010011	000000
$(x_4,0)$	0001	1001	0001
$(x_4,1)$	0001	0110	0000
$(x_5,0)$	01	10	00
$(x_5,1)$	01	01	01

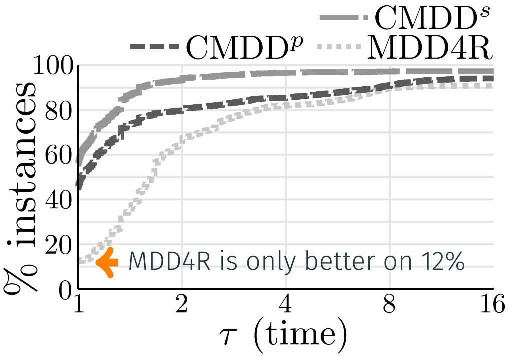


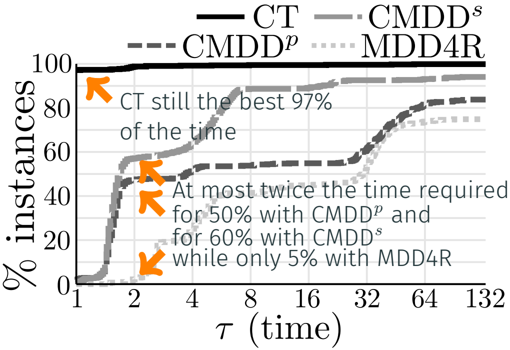
$\Delta_{x_2} = \{1\}$

x_1	x_2	x_3	x_4	x_5
$\{0,1\}$	$\{0\}$	$\{0, \not{1}\}$	$\{0, \not{1}\}$	$\{\not{0}, 1\}$

(x,v)	currArcs[x]	supports[x,v]	$\cap$
$(x_1,0)$	11	10	10
$(x_1,1)$	11	01	01
$(x_3,0)$	001000	101100	001000
$(x_3,1)$	001000	010011	000000
$(x_4,0)$	0001	1001	0001
$(x_4,1)$	0001	0110	0000
$(x_5,0)$	01	10	00
$(x_5,1)$	01	01	01

COMPACT-MDD: RESULTS





CMDD more efficiant than MDD4R

Reduction of gap between table contraint (CT) and layered graph

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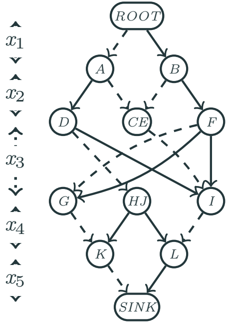
- New type of layered graph (sMDD) allowing **less nodes**
- **More efficiant** layered graph based propagator (CMDD)
- **Gap reduction** between table based (CT) and layered graph based (CMDD) propagator



Thank you for listening!

Any questions?

Feel free to come to my poster (#3777)