

LEARNING OPTIMAL DECISION TREES USING CONSTRAINT PROGRAMMING

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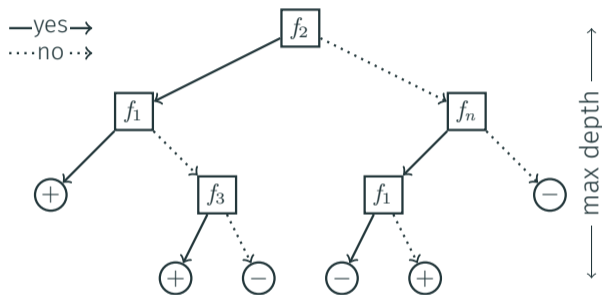
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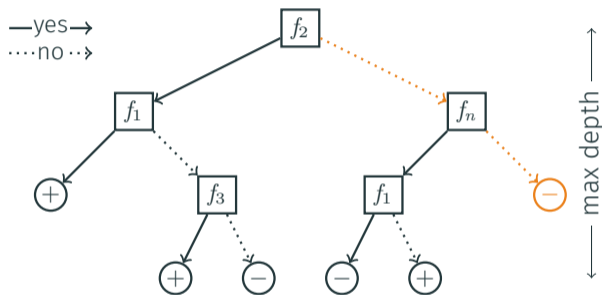
Database					
f_1	f_2	f_3	...	f_n	c
1	0	1	...	1	+
0	1	0	...	1	-
1	1	0	...	0	+
0	0	0	...	0	+
1	0	0	...	0	+
0	1	1	...	1	-
1	1	1	...	0	-
⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	...	1	+



$$\min \sum (pred(i) - c(i))$$

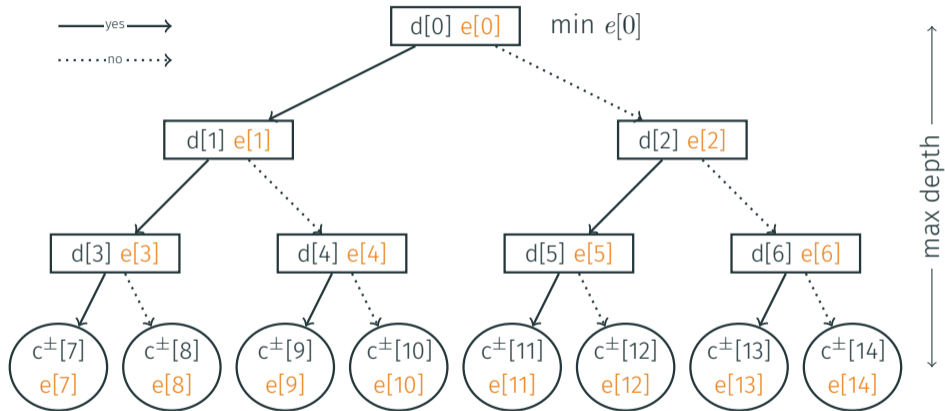
Database					
f_1	f_2	f_3	...	f_n	c
1	0	1	...	1	+
0	1	0	...	1	-
1	1	0	...	0	+
0	0	0	...	0	+
1	0	0	...	0	+
0	1	1	...	1	-
1	1	1	...	0	-
⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	...	1	+

New sample					
0	0	1	...	0	-



$$\min \sum (pred(i) - c(i))$$

- Mining optimal decision trees from itemset lattices, Nijssen, S., Fromont, E., 2007
- Minimising decision tree size as combinatorial optimisation, Bessiere, C., Hebrard, E., O'Sullivan, B., 2009
- Optimal constraint-based decision tree induction from itemset lattices, Nijssen, S., Fromont, É., 2010
- **Optimal classification trees**, Bertsimas, D., Dunn, J., 2017
- Learning optimal decision trees with sat, Narodytska, N., Ignatiev, A., Pereira, F., Marques-Silva, J., RAS, I., 2018
- Learning optimal and fair decision trees for non-discriminative decision-making, Aghaei, S., Azizi, M.J., Vayanos, P., 2019
- Learning optimal classification trees using a binary linear program formulation, Verwer, S., Zhang, Y., 2019



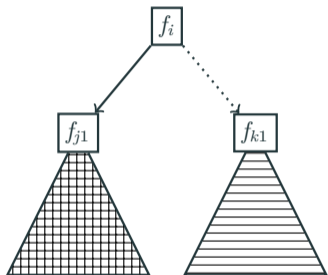
$$\text{dom}(d[i]) = \{0, 1, \dots, n\}$$

$$\begin{aligned} \text{dom}(c^+[i]) &= \{0, \dots, N\} \\ \text{dom}(c^-[i]) &= \{0, \dots, N\} \end{aligned}$$

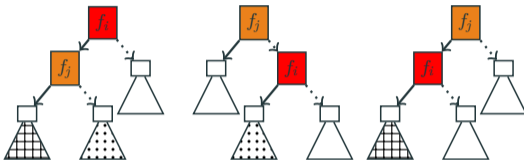
$$\text{dom}(e[i]) = \{0, \dots, N\}$$

Constraints

- enforce the tree structure (AllDifferent, CoverSize,...)
- right computation of the error (allow minimization objective)
- imposing minimum at leaf (additionnal constraint)
- constraints avoiding useless decisions
- redundant constraints (speed improvement)



$$\min T = \min \text{left}(T) + \min \text{right}(T)$$



	yes	no	hash
			f_i, f_j^-
			$f_i - f_j$

	$N_{\min} = 1$			$N_{\min} = 5$			
	DL8	BinOCT	CP	DL8	CP	CP-c	CP-m
Proven optimality	49(64%)	13(17%)	57(75%)	54(71%)	56(74%)	56(74%)	58(76%)
Best solution found	49(64%)	21(28%)	76(100%)	54(71%)	74(97%)	74(97%)	70(92%)
Fastest	23(30%)	11(14%)	49(64%)	28(37%)	40(53%)	33(43%)	22(29%)
Time out	27(36%)	63(83%)	19(25%)	22(29%)	21(28%)	21(28%)	19(25%)

23 instances, depths from 2 to 5, 10 min TO

DL8: Dynamic programming approach using frequent itemsets mining

BinOCT: MIP-based approach running on CPLEX

To summarize

- efficient method
- cp based
- exploits the structure of the problem
- anytime best solution

To go further

- multi-class decision trees
- continuous features through binarization
- other sum-based cost functions
- ...

Thank you for listening!

Any questions?

Also, our extended journal paper is out!

[https://link.springer.com/article/10.1007/
s10601-020-09312-3](https://link.springer.com/article/10.1007/s10601-020-09312-3)