THE EXTENSIONAL CONSTRAINT

ACP dissertation award

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Advisors:

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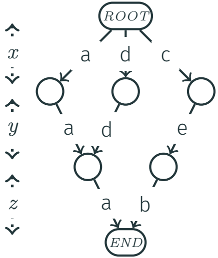
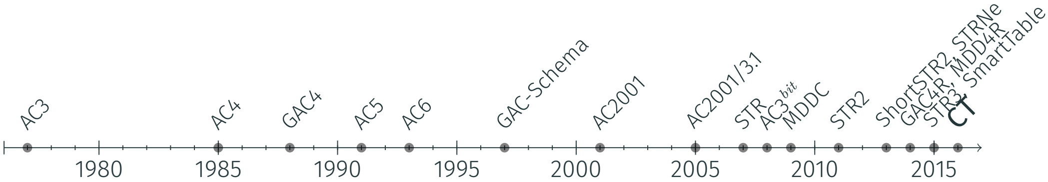
UCLouvain

	x	y	z
τ_1	a	a	a
τ_2	d	d	a
τ_3	c	e	b
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Tables are one of the oldest most used CP constraints

MDDs are equivalent to tables





2016 : New algorithm! Compact-Table [CP2016], based on bitwise operations, completely outperformed existing algorithms

THE 3 DIMENSIONS OF EXTENSION

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CT

State of the art   Published

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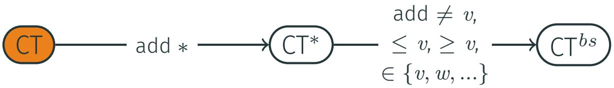
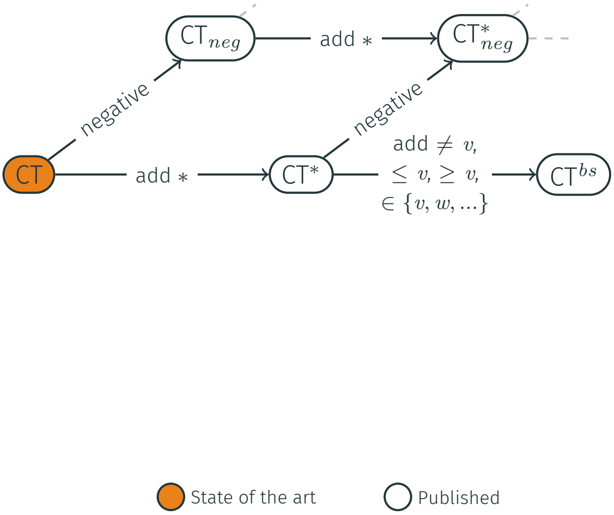


State of the art   Published


CT_{neg}
add $*$
CT^*_{neg}
negative
negative
add $\neq v$,
$\leq v$, $\geq v$,
$\in \{v, w, \ldots\}$
CT
add $*$
CT^*
CT^{bs}
State of the art
Published

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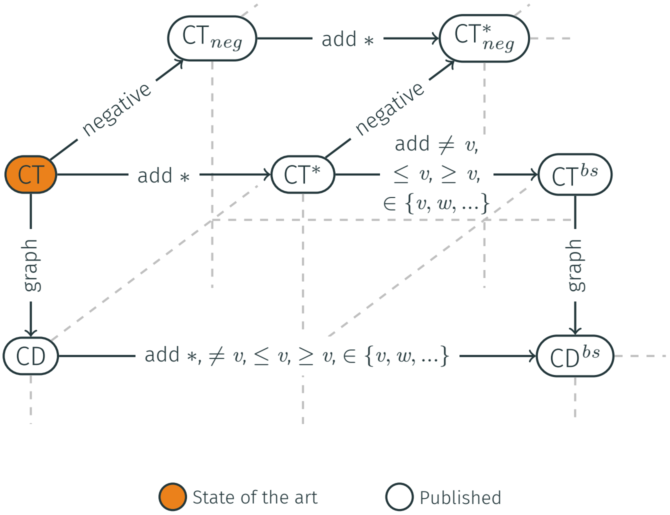

CT_{neg}
add $*$
CT^*_{neg}
negative
negative
CT
add $*$
CT^*
add $\neq v$, $\leq v$, $\geq v$, $\in \{v, w, \ldots\}$
CT^{bs}
graph
graph
CD
add $*, \neq v, \leq v, \geq v, \in \{v, w, \ldots\}$
CD^{bs}
State of the art
Published

COMPACT-TABLE

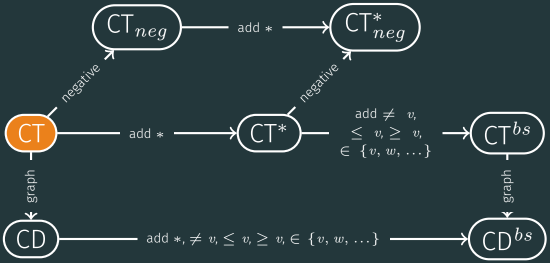

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Table

	x_1	x_2	x_3
τ_1	a	c	a
τ_2	b	b	b
τ_3	a	c	b
τ_4	c	a	b
τ_5	b	c	b
τ_6	c	b	c
τ_7	a	a	b
τ_8	b	b	c

`currTable`

τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8
1	1	1	1	1	1	1	1

Reversible Sparse Bitset

`supports`

	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8
$[x_1, a]$	1	0	1	0	0	0	1	0
$[x_1, b]$	0	1	0	0	1	0	0	1
$[x_1, c]$	0	0	0	1	0	1	0	0
$[x_2, a]$	0	0	0	1	0	0	1	0
$[x_2, b]$	0	1	0	0	0	1	0	1
$[x_2, c]$	1	0	1	0	1	0	0	0
$[x_3, a]$	1	0	0	0	0	0	0	0
$[x_3, b]$	0	1	1	1	1	0	1	0
$[x_3, c]$	0	0	0	0	0	1	0	1

Precomputed Bitsets

COMPACT-TABLE: DATA STRUCTURES

UCLouvain

Table

	x_1	x_2	x_3
τ_1	a	c	a
τ_2	b	b	b
τ_3	a	c	b
τ_4	c	a	b
τ_5	b	c	b
τ_6	c	b	c
τ_7	a	a	b
τ_8	b	b	c

currTable (Reversible Sparse Bitset)

	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8
currTable	1	1	1	1	1	1	1	1

supports (Precomputed Bitsets)

	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8
$[x_1, a]$	1	0	1	0	0	0	1	0
$[x_1, b]$	0	1	0	0	1	0	0	1
$[x_1, c]$	0	0	0	1	0	1	0	0
$[x_2, a]$	0	0	0	1	0	0	1	0
$[x_2, b]$	0	1	0	0	0	1	0	1
$[x_2, c]$	1	0	1	0	1	0	0	0
$[x_3, a]$	1	0	0	0	0	0	0	0
$[x_3, b]$	0	1	1	1	1	0	1	0
$[x_3, c]$	0	0	0	0	0	1	0	1

Classical update

Δ_x = { supports[x,b], supports[x,d], supports[x,f] }

supports[x,b]	0	0	0	1
supports[x,d]	1	0	0	0
supports[x,f]	0	1	0	0

$\sim \cup =$

mask	0	0	1	0

$\cap$

old currTable	1	1	1	0

$=$

new currTable	0	0	1	0

Reset update

$dom(x)$ = { supports[x,a], supports[x,c], supports[x,e] }

supports[x,a]	1	0	0	0
supports[x,c]	0	1	0	0
supports[x,e]	0	0	0	1

$\cup =$

mask	1	1	0	1

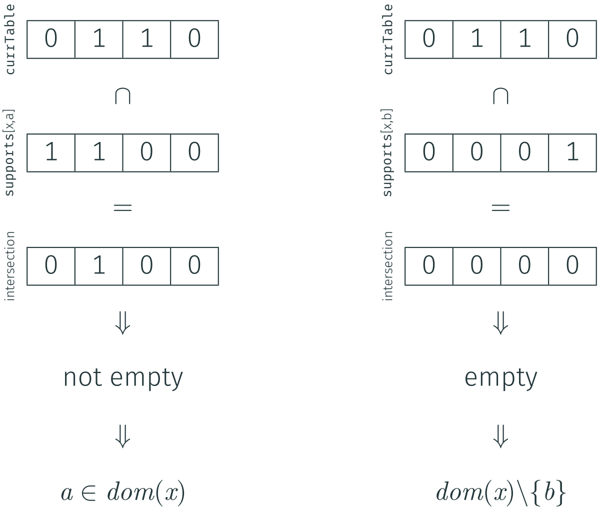
$\cap$

old currTable	1	0	1	0

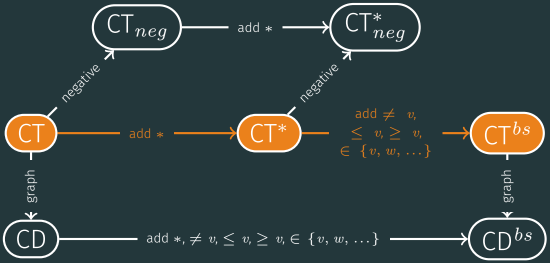
$=$

new currTable	1	0	0	0

CT: PROPAGATION



1ST DIMENSION: FROM GROUND TABLES TO SMART TABLES



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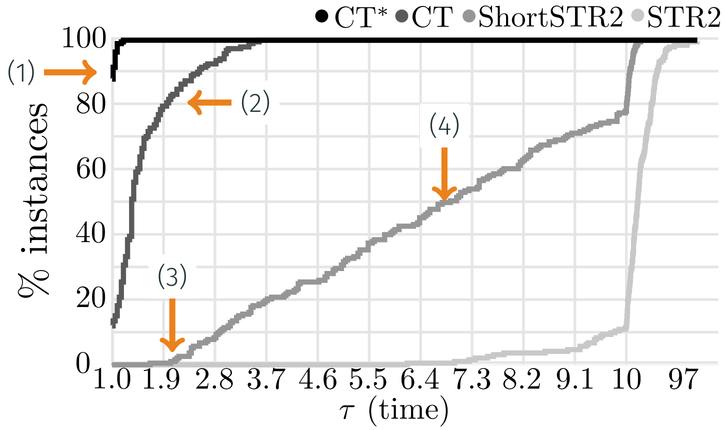
A Basic Smart Table — contains unary Smart Elements — representing multiples values

	x	y	z
τ_1	$*$	$*$	$\in \{a, b\}$
τ_2	$\neq a$	c	$\leq a$
τ_3	b	$*$	$*$
τ_4	$\geq c$	$\neq b$	a
	$\vdots$	$\vdots$	$\vdots$

- single value: e — ~~a~~, ~~b~~, ~~c~~, ~~d~~, e, ~~f~~
- universal value: $*$ — a, b, c, d, e, f
- exclusion: $\neq e$ — a, b, c, d, ~~e~~, f
- upper bound: $\leq c$ — a, b, c, ~~d~~, ~~e~~, ~~f~~
- lower bound: $\geq c$ — ~~a~~, ~~b~~, c, d, e, f
- set: $\in \{a, c, d\}$ — a, ~~b~~, c, d, ~~e~~, ~~f~~

$*$ ↓

Δ^x	supports*$[x,b]$	0	0	0	1
	supports*$[x,d]$	0	0	0	0
	supports*$[x,f]$	0	1	0	0
			$\sim \cup =$		
	mask	1	0	1	0
			$\cap$		
	old currTable	1	1	1	0
			$=$		
	new currTable	1	0	1	0



Complexity of CT*:
same as CT ($\mathcal{O}(rd\frac{t}{w})$)

(1) CT* best 90% of the time

(2) CT requires $< 2\times$ time on 80%

(3) ShortSTR2 needs $> 2\times$ time

(4) ShortSTR2 needs $> 7\times$ time on 50%

600 instances, 20 variables, domain size from 5 to 7, 40 random tables by instances, arity of 6 or 7, tightness [0.5%;2%], 1, 5, 10 or 20 % of short tuples

$|dom(x)| == 0$

Trivial!
Handled by variable x

$|dom(x)| == 1$

$|\Delta_x| \geq |dom(x)|$ always true!
`ResetUpdate(x)` used
and already working!

$|dom(x)| > 1$

If $|\Delta_x| < |dom(x)|$

Tuple always valid!
At least one valid value
`supports`$^*[x][\tau] = 0$

If $|\Delta_x| \geq |dom(x)|$

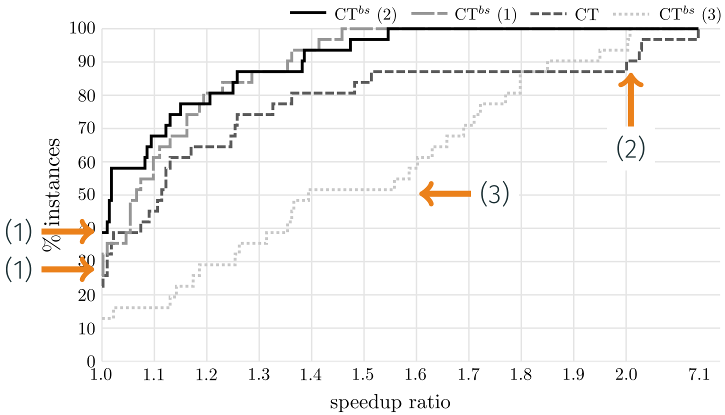
`ResetUpdate(x)` used
and already working!

ADDITION OF $\leq v$ AND $\geq v$

	$\leq d$	$\leq b$	$\geq e$	$\geq b$
$\sim$`mask`	1	1	1	1
$\cap$				
`supportsMin`$[x, \underbrace{c}_{x.min}]$	1	0	1	1
$\cap$				
`supportsMax`$[x, \underbrace{d}_{x.max}]$	1	1	0	1
$=$				
new `mask`	1	0	0	1
$\cap$				
old `currTable`	0	0	1	1
$=$				
new `currTable`	0	0	0	1

$\|dom(x)\|$	sets		structured sets	
1	$\{a\}$	1	$*$	1
2	$\{a\}, \{b\}, \{a, b\}$	3	$a, b, *$	3
3	$\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$	7	$a, b, c, \neq a, \neq b, \neq c, *$	7
4	$\{a\}, \{b\}, \ldots, \{a, b\}$, {a,c}, {a,d}, {b,c}, {b,d}, $\{c, d\}, \{a, b, c\}, \ldots, \{a, b, c, d\}$	15	$a, b, c, d, \leq b, \geq c, \neq a, \neq b, \neq c, \neq d, *$	11
5	$\{a\}, \{b\}, \ldots, \{a, b\}$, {a,c}, {a,d}, {a,e}, {b,c}, {b,d}, {b,e}, {c,d}, {c,e}, $\{a, b, c\}$, {a,b,d}, {a,b,e}, {a,c,d}, {a,c,e}, $\ldots, \{a, b, c, d\}, \ldots, \{a, b, c, d, e\}$	31	$a, b, c, d, e, \leq b, \leq c, \geq c, \geq d, \neq a, \neq b, \neq c, \neq d, \neq e, *$	15

- `supports`$[x,v]$: supports value v
- `supports`$^*[x,v]$: supports only value v
- `supportsMin`$[x,v]$: supports at least one value $\geq v$
- `supportsMax`$[x,v]$: supports at least one value $\leq v$

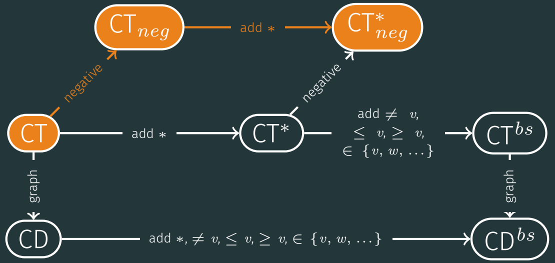


Complexity of CT^{bs}:
same as CT ($\mathcal{O}(rd\frac{t}{w})$)

(1) CT^{bs} best on 40 + 30%
(2) CT needs $< 2\times$ for 88%
(3) Overhead due to Set only

XCSP3 instances with only tables, transformed into basic smart table with at least 10% compression (1) with only $\leq v$ and $\geq v$, (2) with $\leq v$ and $\geq v$ + post processing to add $*$ and $\neq v$, (3) with elements treated as simple sets

2ND DIMENSION: FROM POSITIVE TO NEGATIVE TABLES



CT_{neg}: SAME DATA STRUCTURES AS CT WITH DIFFERENT MEANINGS

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Negative Table

	x_1	x_2	x_3
τ_1	a	c	a
τ_2	b	b	b
τ_3	a	c	b
τ_4	c	a	b
τ_5	b	c	b
τ_6	c	b	c
τ_7	a	a	b
τ_8	b	b	c

currTable (Reversible Sparse Bitset)

	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8
currTable	1	1	1	1	1	1	1	1

dangerous supports (Precomputed Bitsets)

	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8
$[x_1, a]$	1	0	1	0	0	0	1	0
$[x_1, b]$	0	1	0	0	1	0	0	1
$[x_1, c]$	0	0	0	1	0	1	0	0
$[x_2, a]$	0	0	0	1	0	0	1	0
$[x_2, b]$	0	1	0	0	0	1	0	1
$[x_2, c]$	1	0	1	0	1	0	0	0
$[x_3, a]$	1	0	0	0	0	0	0	0
$[x_3, b]$	0	1	1	1	1	0	1	0
$[x_3, c]$	0	0	0	0	0	1	0	1

Hypothesis

No duplicate!

No overlap!

CT_{neg}: CT WITH MODIFIED PROPAGATION

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currTable	1	1	1	0
		$\cap$		
supports[x,a]	0	0	1	1
		$=$		
intersection	0	0	1	0

$$\Downarrow$$

$$\texttt{bitCount} = 1 \neq \underbrace{|dom(y)|}_{=2} \times \underbrace{|dom(z)|}_{=1}$$

$$\Downarrow$$

$$a \in dom(x)$$

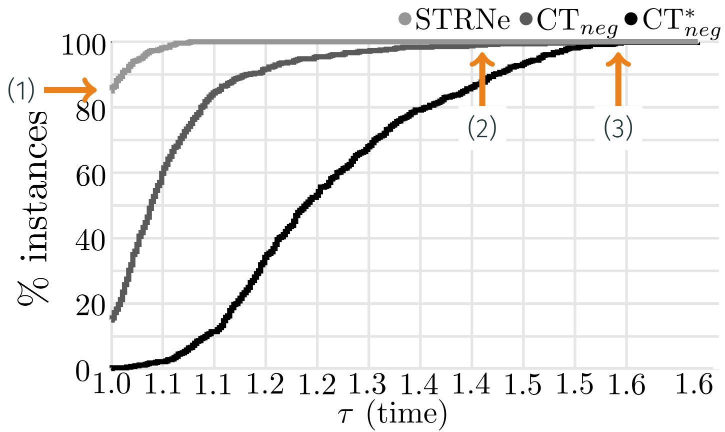
currTable	1	1	1	0
		$\cap$		
supports[x,b]	1	1	0	0
		$=$		
intersection	1	1	0	0

$$\Downarrow$$

$$\texttt{bitCount} = 2 = \underbrace{|dom(y)|}_{=2} \times \underbrace{|dom(z)|}_{=1}$$

$$\Downarrow$$

$$dom(x) \backslash \{b\}$$



Complexity of CT_{neg}:
CT's complexity × complexity of bitcount ($\mathcal{O}(rd\frac{t}{w}k)$)

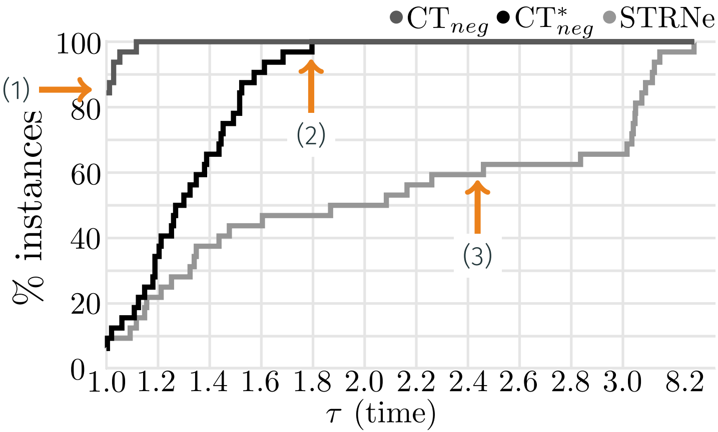
Complexity of CT^*_{neg}:
CT_{neg}'s complexity, using t' the # of tuples with dummy ones ($\mathcal{O}(rd\frac{t'}{w}k)$)

(1) STRNe best 85%

(2) CT_{neg} requiers max $1.4\times$

(3) CT_{neg} requiers max $1.6\times$

600 instances (with high number of solution), 20 variables, domain size from 5 to 7, 40 random tables by instances, arity of 6 or 7, tightness [0.5%;2%], 1, 5, 10 or 20 % of short tuples



Complexity of CT_{neg}: CT's complexity $\times$ complexity of bitcount $(\mathcal{O}(rd\frac{t}{w}k))$

Complexity of CT^*_{neg}: CT_{neg}'s complexity, using t' the # of tuples with dummy ones $(\mathcal{O}(rd\frac{t'}{w}k))$

(1) CT_{neg} best 85% of the time

(2) $< 1.8\times$ for CT^*_{neg}

(3) STRNe requiers $> 2.5\times$ on 40% of instances

45 instances (with low number of solutions), 10 variables, domain size of 5, 40 random tables by instances, arity of 6, tightness 10,... 90%, no short tuples

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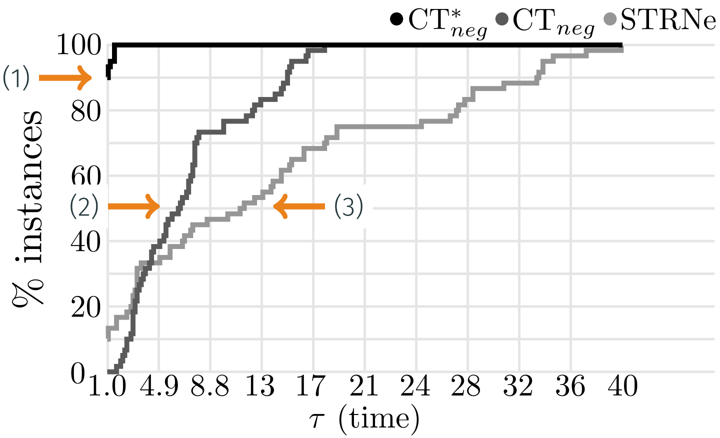


Complexity of CT_{neg}: CT's complexity × complexity of bitcount ($\mathcal{O}(rd\frac{t}{w}k)$)

Complexity of CT^*_{neg}: CT_{neg}'s complexity, using t' the # of tuples with dummy ones ($\mathcal{O}(rd\frac{t'}{w}k)$)

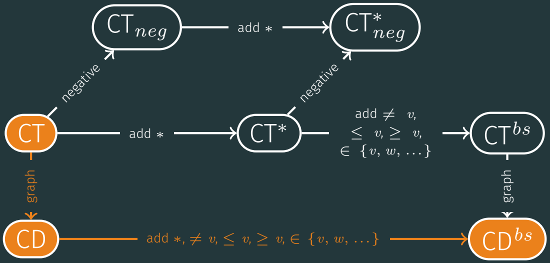
(1) CT^*_{neg} best 90% of the time

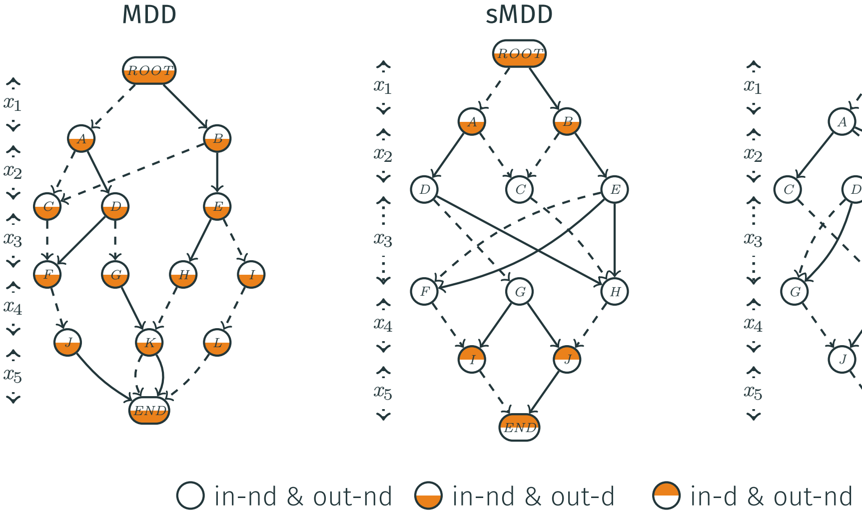
(2) $> 6\times$ for 50%

(3) $> 11\times$ for 50%

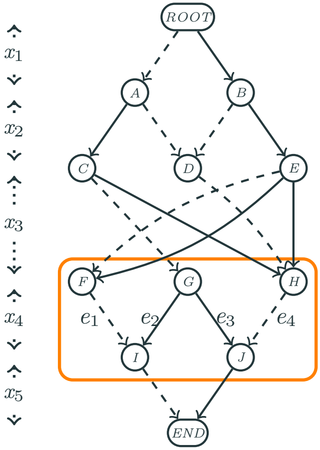
100 instances (with low number of solutions), 3 variables, domain size of 100, 40 random tables by instances, arity of 3, tightness [0.5;2%], 5, 10 or 20 % of short tuples

3RD DIMENSION: FROM TABLES TO GRAPHS

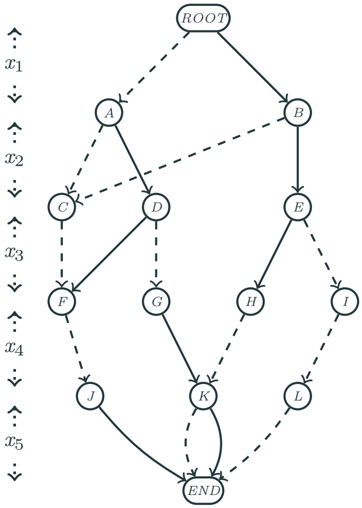



MDD
sMDD
MVD
ROOT
A
B
C
D
E
F
G
H
I
J
K
L
END
x_1
x_2
x_3
x_4
x_5
in-nd & out-nd
in-nd & out-d
in-d & out-nd

DATA STRUCTURE



Name	Set	Bit-set
`currArcs`$[x_4]$	$\{e_1, e_2, e_3, e_4\}$	[1 1 1 1]
`supports`$[x_4,0]$	$\{e_1, \cancel{e_2}, \cancel{e_3}, e_4\}$	[1 0 0 1]
`arcsT`$[G,x_4]$	$\{\cancel{e_1}, e_2, e_3, \cancel{e_4}\}$	[0 1 1 0]
`arcsH`$[x_4,I]$	$\{e_1, e_2, \cancel{e_3}, \cancel{e_4}\}$	[1 1 0 0]



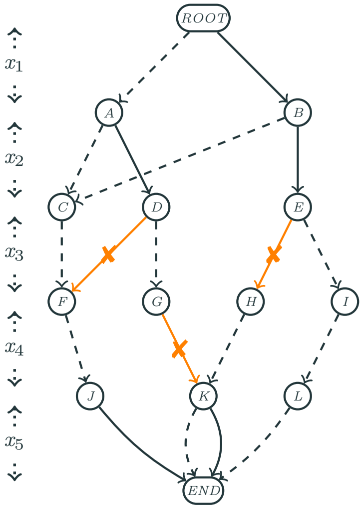
currArcs[x_1]
[1 1]

currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 1 1 1 1]

currArcs[x_4]
[1 1 1 1]

currArcs[x_5]
[1 1 1 1]



$\texttt{currArcs}[x_1]$
[1 1]

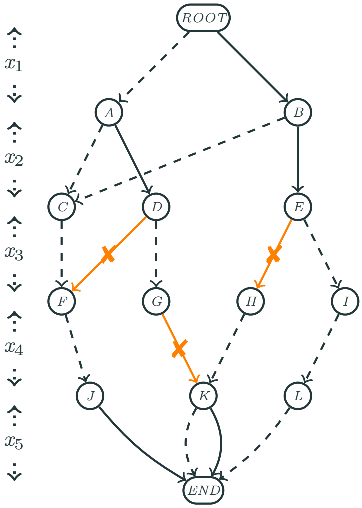
$\texttt{currArcs}[x_2]$
[1 1 1 1]

$\texttt{currArcs}[x_3]$
[1 1 1 1 1]

$\texttt{currArcs}[x_4]$
[1 1 1 1]

$\texttt{currArcs}[x_5]$
[1 1 1 1]

1st step
Direct removal



currArcs[x_1]
[1 1]

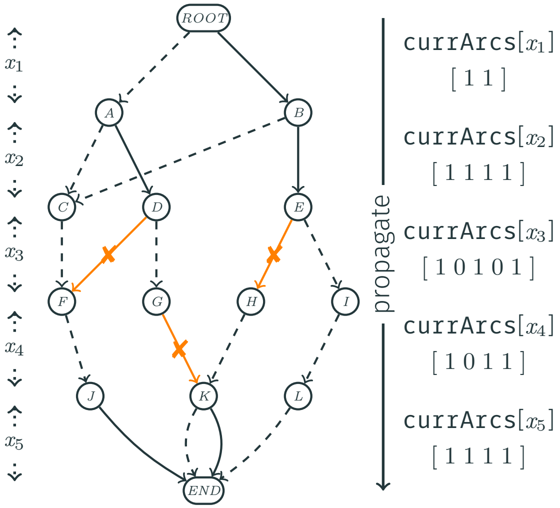
currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 0 1]

currArcs[x_4]
[1 0 1 1]

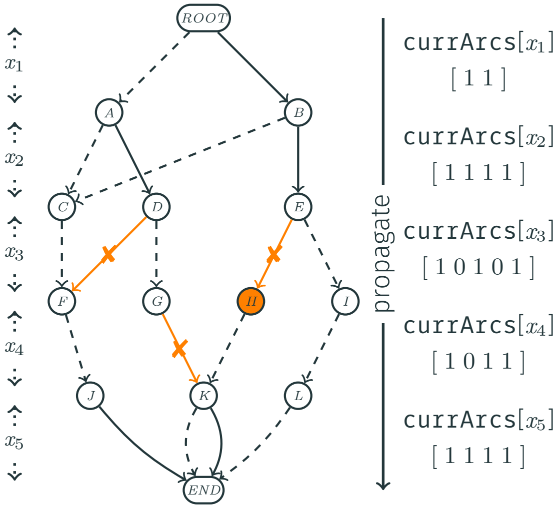
currArcs[x_5]
[1 1 1 1]

1st step
Direct removal



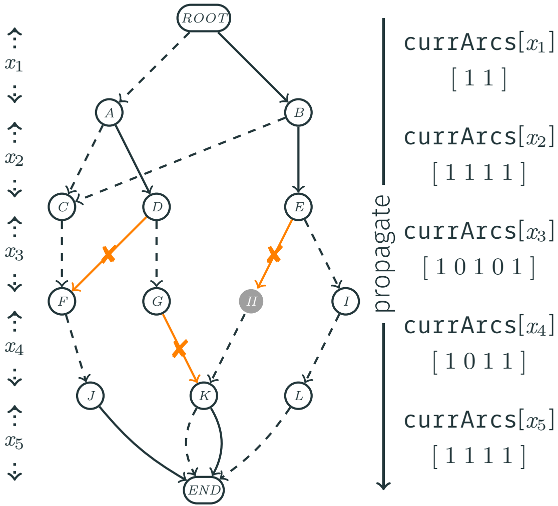
2nd step

Top down

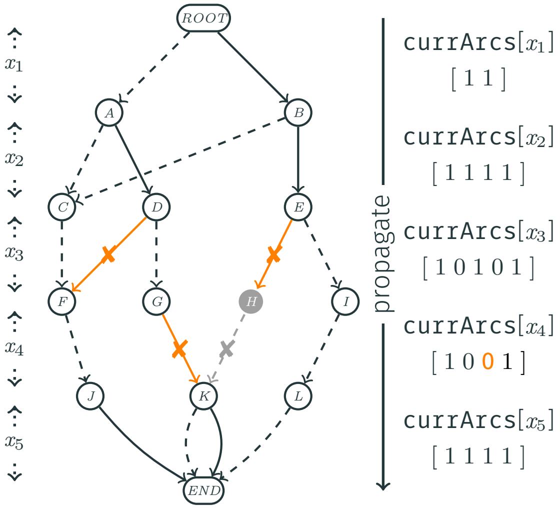


2nd step
Top down

COMPACT-DIAGRAM: UPDATE



2nd step
Top down



2nd step
Top down

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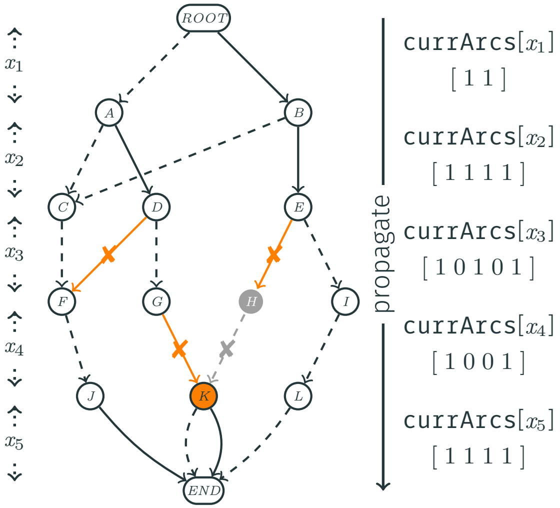
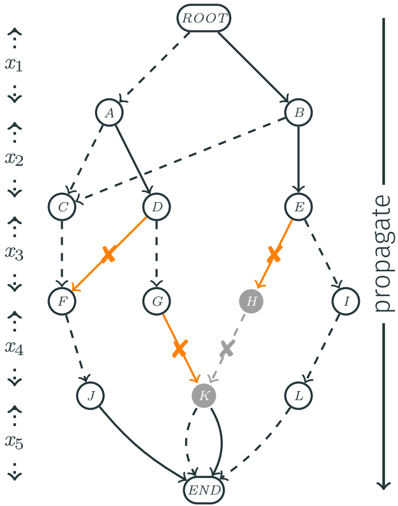


2nd step
Top down



currArcs[x_1]
[1 1]

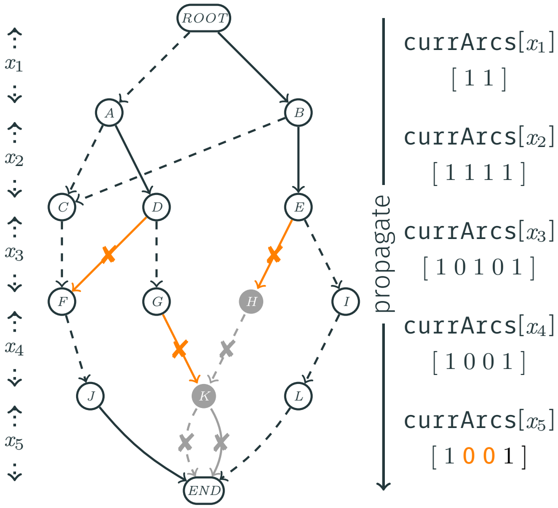
currArcs[x_2]
[1 1 1 1]

currArcs[x_3]
[1 0 1 0 1]

currArcs[x_4]
[1 0 0 1]

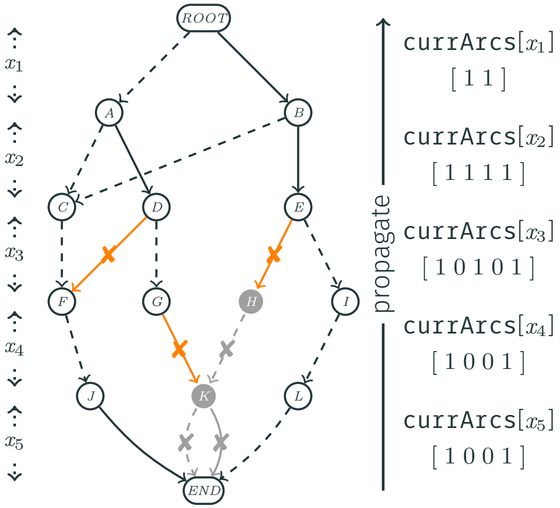
currArcs[x_5]
[1 1 1 1]

2nd step
Top down

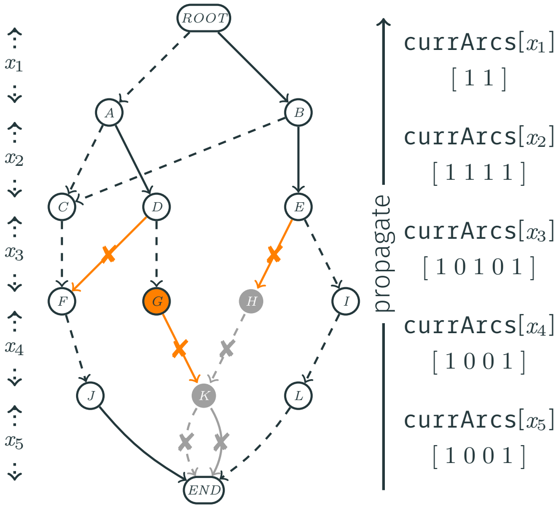


2nd step

Top down



3rd step
Bottom up

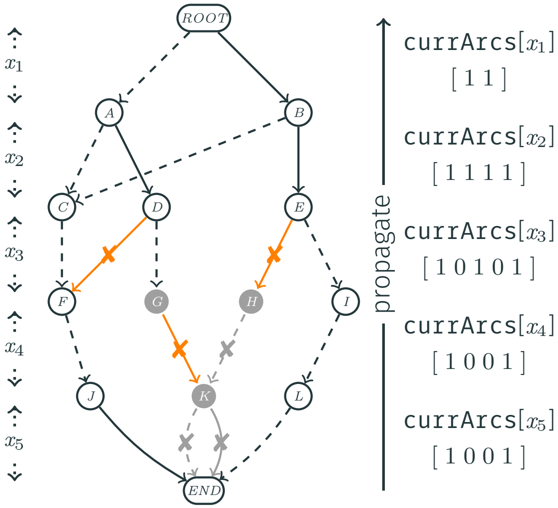


3rd step

Bottom up

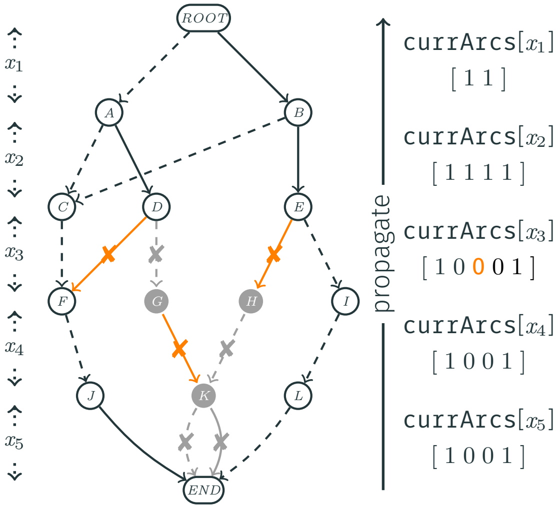
COMPACT-DIAGRAM: UPDATE

UCLouvain



3rd step
Bottom up

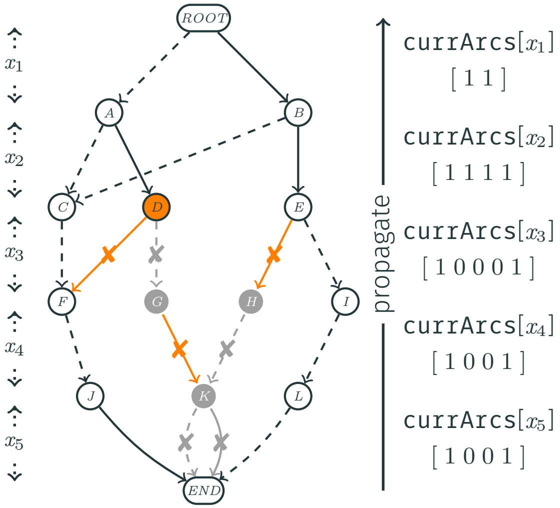
COMPACT-DIAGRAM: UPDATE



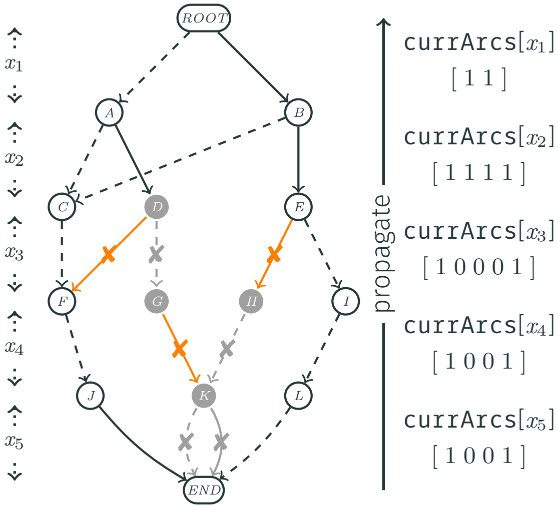
3rd step

Bottom up

COMPACT-DIAGRAM: UPDATE



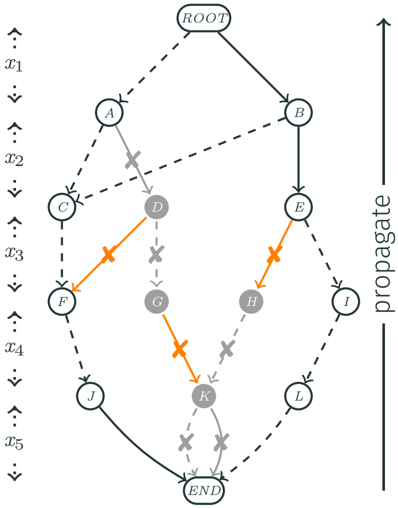
3rd step
Bottom up



3rd step

Bottom up

UCLouvain



currArcs[x_1]
[1 1]

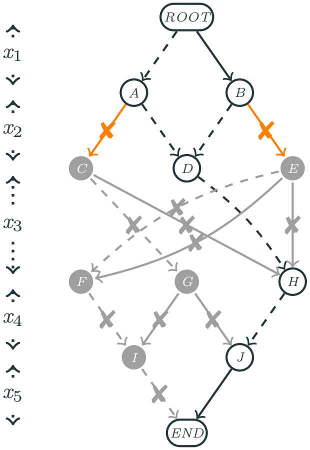
currArcs[x_2]
[1 0 1 1]

currArcs[x_3]
[1 0 0 0 1]

currArcs[x_4]
[1 0 0 1]

currArcs[x_5]
[1 0 0 1]

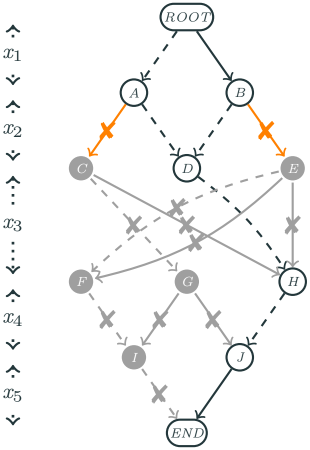
3rd step
Bottom up



$\Delta_{x_2} = \{1\}$

x_1	x_2	x_3	x_4	x_5
$\{0,1\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$

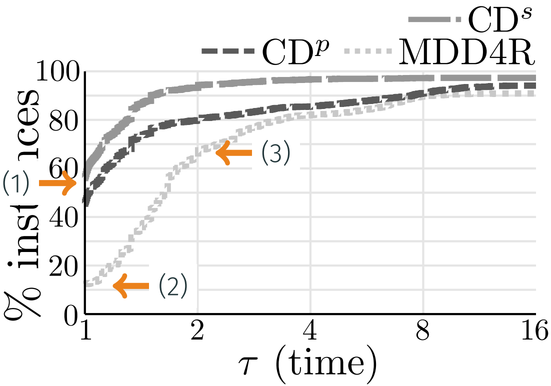
(x,v)	`currArcs`[x]	`supports`[x,v]	$\cap$
$(x_1,0)$	11	10	10
$(x_1,1)$	11	01	01
$(x_3,0)$	001000	101100	001000
$(x_3,1)$	001000	010011	000000
$(x_4,0)$	0001	1001	0001
$(x_4,1)$	0001	0110	0000
$(x_5,0)$	01	10	00
$(x_5,1)$	01	01	01



$\Delta_{x_2} = \{1\}$

x_1	x_2	x_3	x_4	x_5
$\{0,1\}$	$\{0\}$	$\{0, \not{1}\}$	$\{0, \not{1}\}$	$\{\not{0}, 1\}$

(x,v)	currArcs[x]	supports[x,v]	$\cap$
$(x_1,0)$	11	10	10
$(x_1,1)$	11	01	01
$(x_3,0)$	001000	101100	001000
$(x_3,1)$	001000	010011	000000
$(x_4,0)$	0001	1001	0001
$(x_4,1)$	0001	0110	0000
$(x_5,0)$	01	10	00
$(x_5,1)$	01	01	01



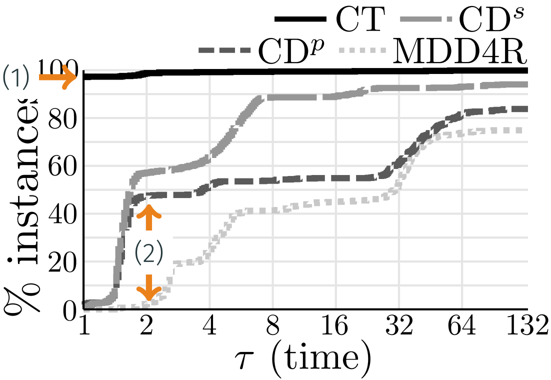
Complexity of CD: similar to CT ($\mathcal{O}(\max(n, d) r \frac{a}{w})$)

(1) CD gives best results, sMDDs better than MDDs

(2) MDD4R only best on 12%

(3) MDD4R requires $> 2\times$ on 35%

XCSP3 instances with only tables, transformed into sMDD or MDD instances only



Complexity of CD: similar to CT ($\mathcal{O}(\max(n, d) r \frac{a}{w})$)

(1) CT still best 95%

(2) Reduction of the gap: CD^s requires $< 2\times$ for 60%, CD^p requires $< 2\times$ for 50%, while MDD4R requires $< 2\times$ for 5%

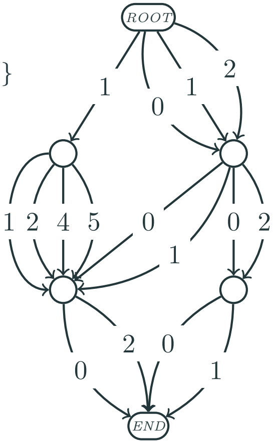
XCSP3 instances with only tables, transformed into sMDD or MDD instances only

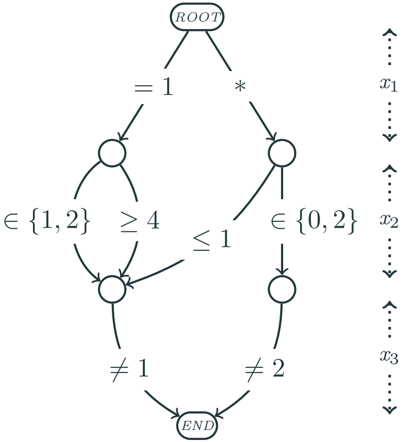
Domains

$x_0 : \{0, 1, 2\}$

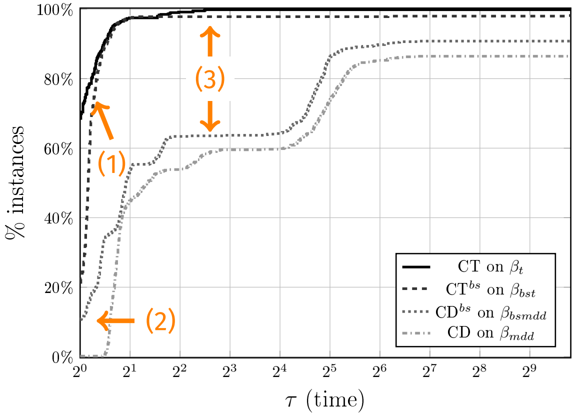
$x_1 : \{0, 1, 2, 3, 4, 5\}$

$x_2 : \{0, 1, 2\}$





UCLouvain



(1) CT and CT^{bs} still dominating

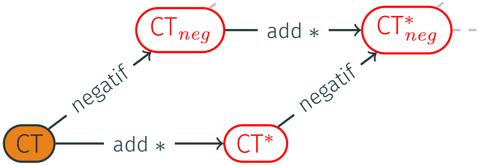
(2) CD^{bs} becomes efficient when compression is high

(3) gap reduced

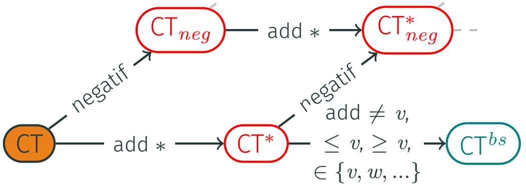
XCSP3 instances with only tables, transformed into bs-table, MDD and bs-MDD instances only

CONCLUSION

CT

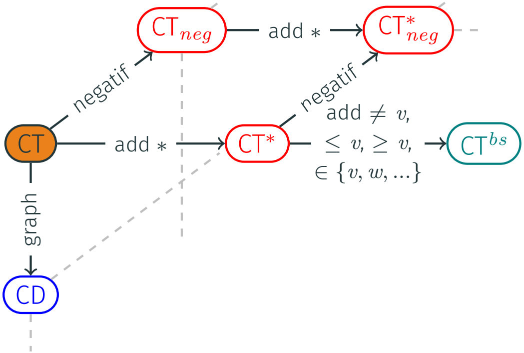


H. Verhaeghe, C. Lecoutre and P. Schaus. **Extending Compact-Table to Negative and Short Tables.** AAAI17



H. Verhaeghe, C. Lecoutre and P. Schaus. **Extending Compact-Table to Negative and Short Tables.** AAAI17

H. Verhaeghe, C. Lecoutre, Y. Deville and P. Schaus. **Extending Compact-Table to Basic Smart Tables.** CP2017



H. Verhaeghe, C. Lecoutre and P. Schaus. **Extending Compact-Table to Negative and Short Tables.** AAAI17
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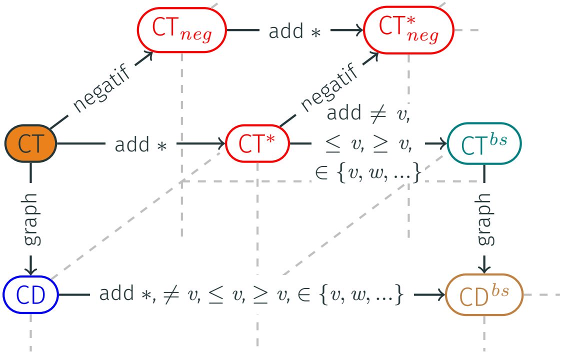
H. Verhaeghe, C. Lecoutre, P. Schaus. **Compact-MDD: Efficiently filtering (s)mdd constraints with Reversible Sparse Bit-Sets.** IJCAI18



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H. Verhaeghe, C. Lecoutre, P. Schaus. **Compact-MDD: Efficiently filtering (s)mdd constraints with Reversible Sparse Bit-Sets.** IJCAI18

H. Verhaeghe, C. Lecoutre, P. Schaus. **Extending Compact-Diagram to Basic Smart Multi-Valued Variable Diagrams.** CPAIOR19

- Increasing non-determinism in diagrams
- Closing the gap between diagrams and tables propagators
- Direct use of compressed tables and non-deterministic diagrams in applications

Thank you for listening!

Any questions?

`https://hverhaeghe.bitbucket.io/`