BIENVENUE À OUIDAH !

NOUS SOMMES RAVIS DE VOUS ACCUEILLIR À **L'ACP SUMMER SCHOOL 2025**. CINQ JOURS D'ÉCHANGES PASSIONNANTS ET DE DÉCOUVERTES VOUS ATTENDENT DANS CE CADRE UNIQUE.

ENJOY YOUR STAY !

25-29 août 2025 / IRSP, Ouidah, Bénin

ACP SUMMER SCHOOL 2025

Fundamentals of Optimisation and Introduction to Constraint Programming

Hélène Verhaeghe

ACP

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Mens Molem Agitat UNIVERSITÉ D'ABOMEY-CALAVI

Artificial Intelligence

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A FIELD OF ARTIFICIAL INTELLIGENCE

Artificial Intelligence

Natural Language Processing

Machine Learning

Intelligent Robots

Constraint Satisfaction and Optimisation

Game Theory

Data Mining

Multiagent Systems

Knowledge Representation

Computer Vision

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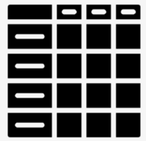

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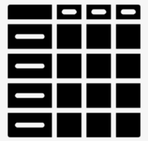

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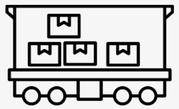

- Box packing: Given a rectangular container, and rectangular boxes. Can you fit the boxes in the container? If yes, how?

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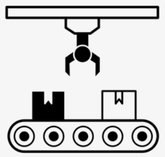

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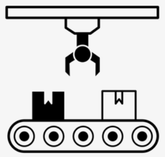

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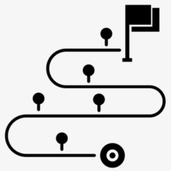

- Traveling Salesman Problem: Given N cities with given distance between pairs of cities, what is the smallest path to visit each cities and then come back to the starting point?

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VARIOUS PARADIGMS FOR VARIOUS GOALS

	Complete exploration	Incomplete exploration
Methods	•Branch and bounds •**Constraint programming** •Integer programming •SAT solving •…	•Local search •Large Neighbourhood Search (LNS) •Genetic Algorithms •Meta-heuristics •…
Pros	Optimality proof	Aim to quickly find good solutions
Cons	Takes time to proof optimality	No Optimality proof

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- Checking all the possible solutions one by one is not tractable
- Require intelligent exploration of the solution space to find good solutions

HOLY GRAIL OF PROGRAMMING

"Constraint programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming: the user states the problem, the computer solves it" - E. Freuder (1996)

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Declarative programming:

- The user describes what the program must accomplish, rather than describing how to accomplish it

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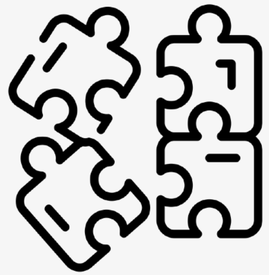
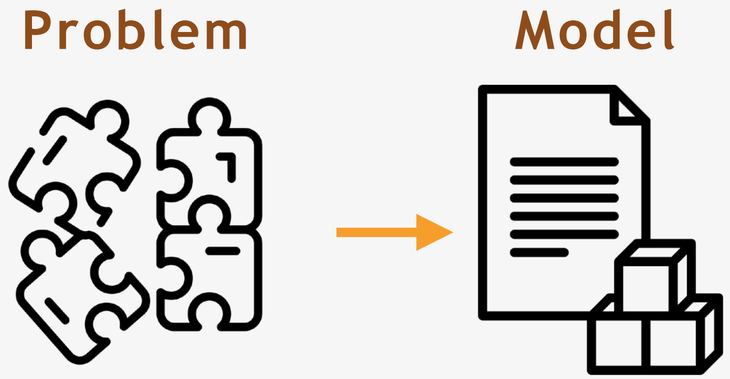
A CP Model Is a Declarative Description of the Solution

Problem



THE CONSTRAINT PROGRAMMING PARADIGM



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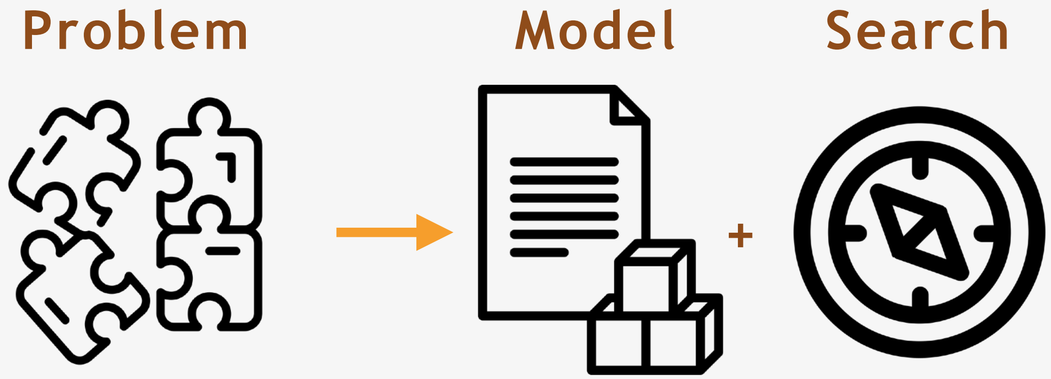

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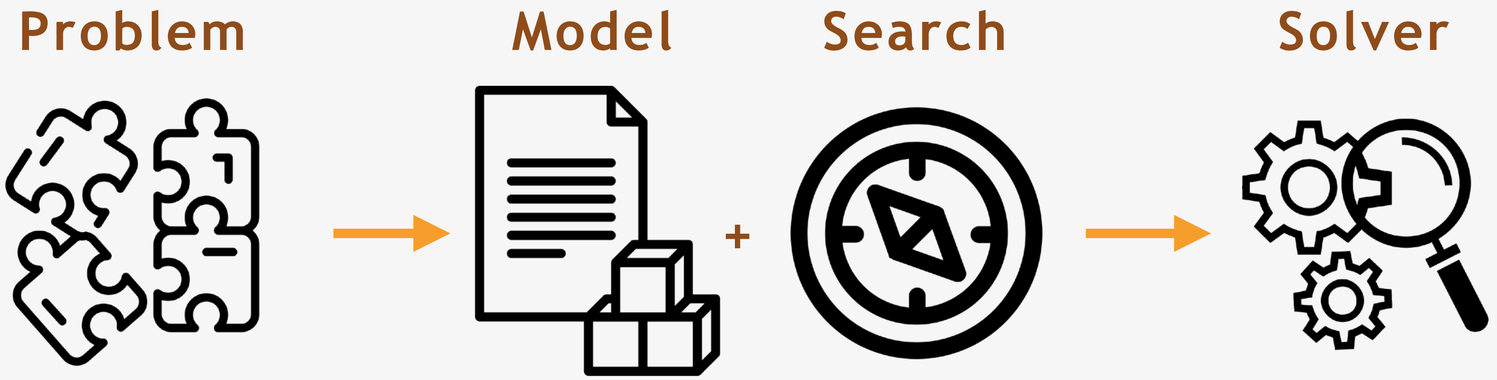

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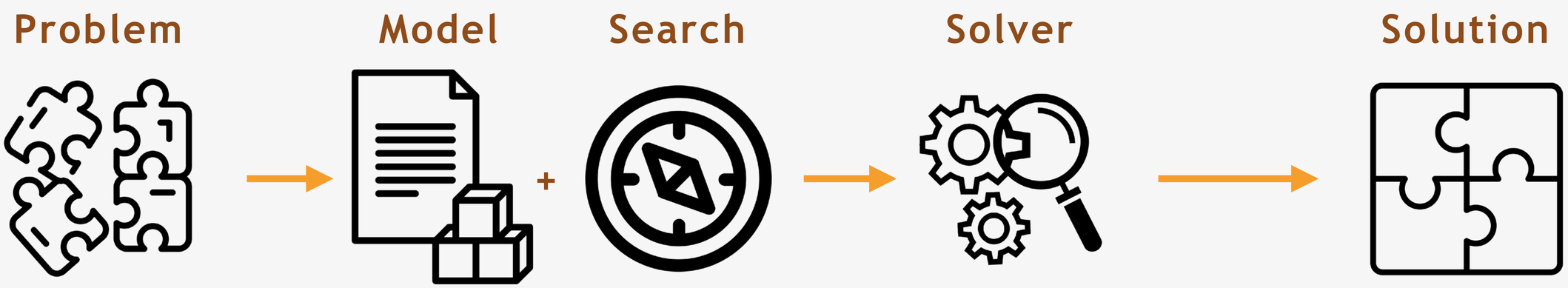

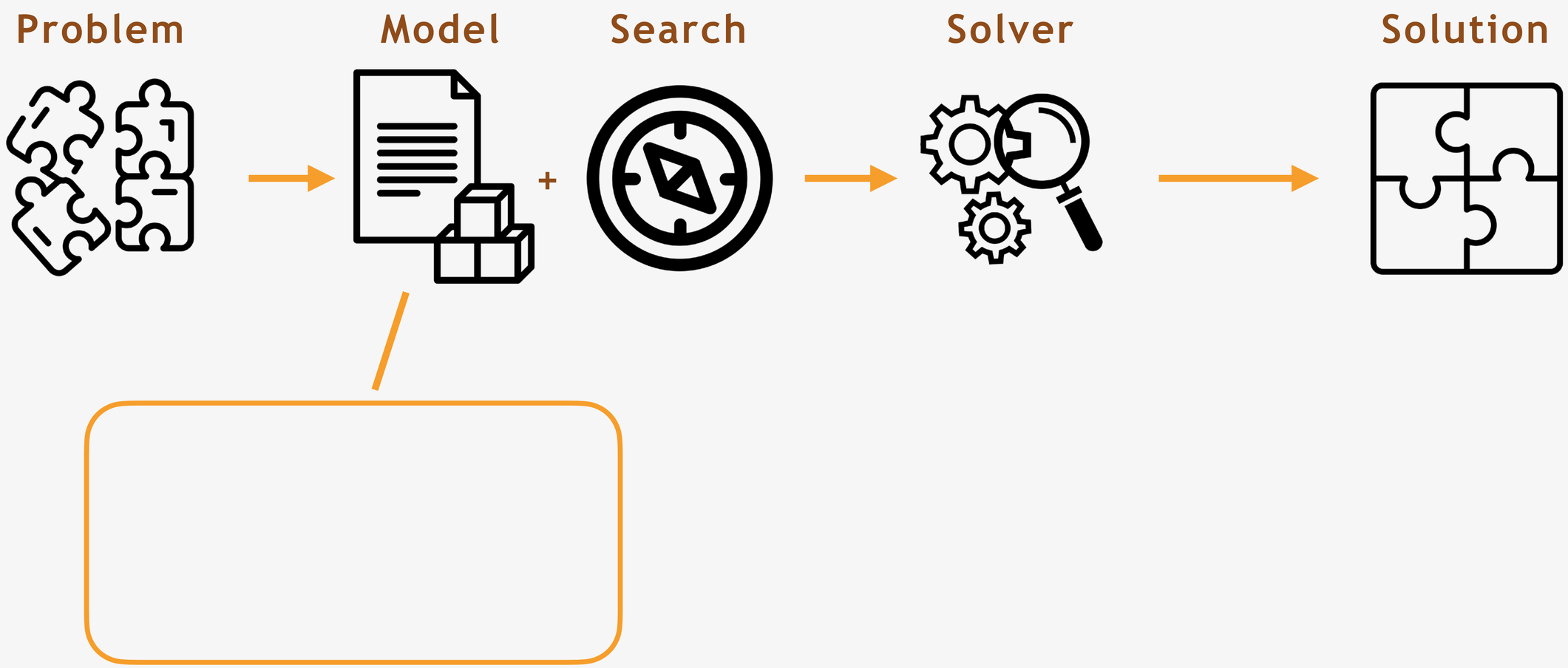
Problem
Model
Search
Solver
Solution
+

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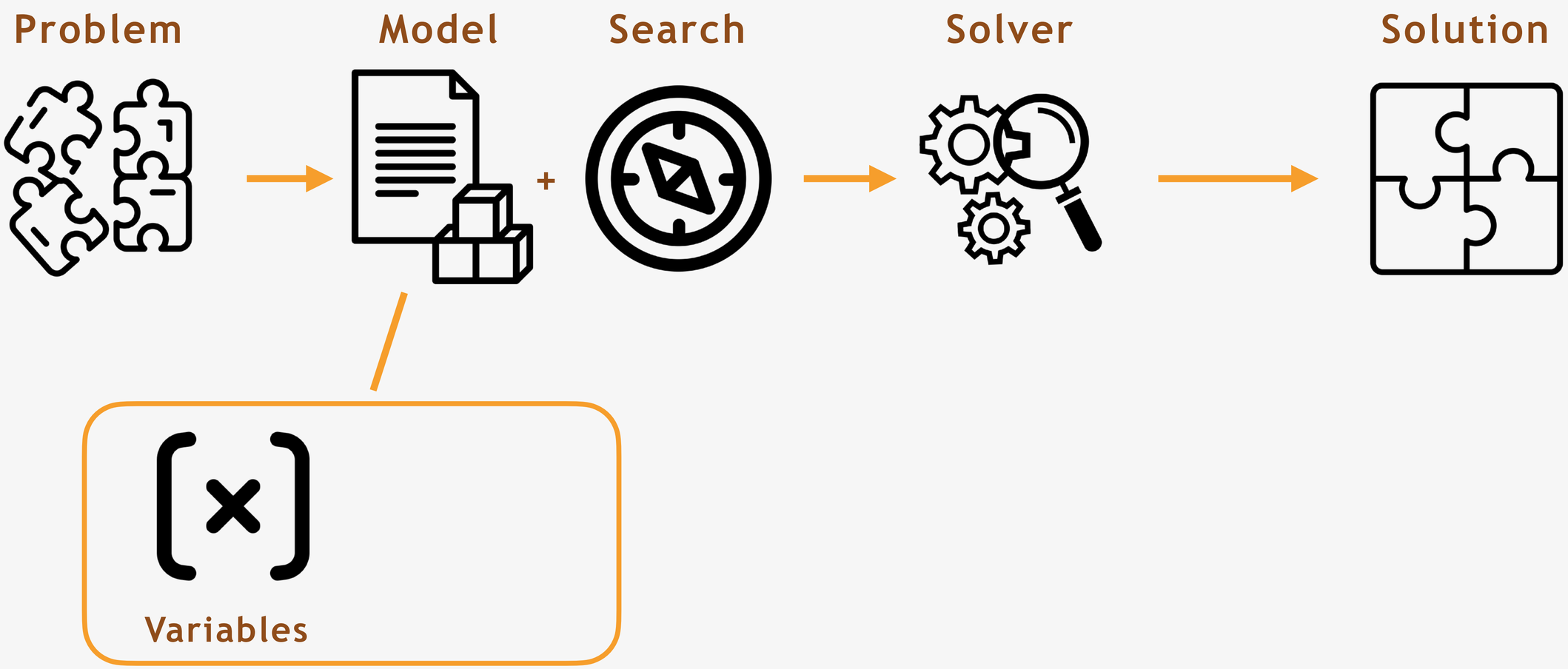

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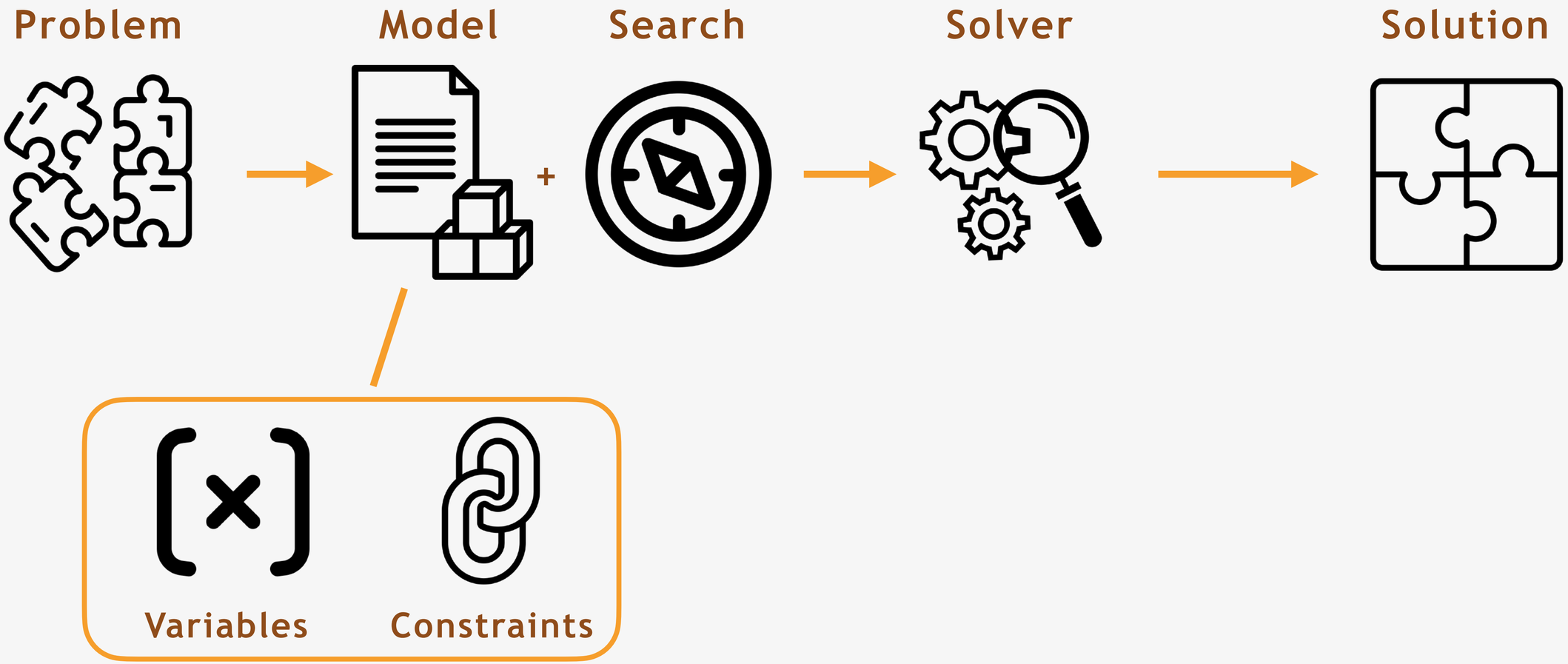

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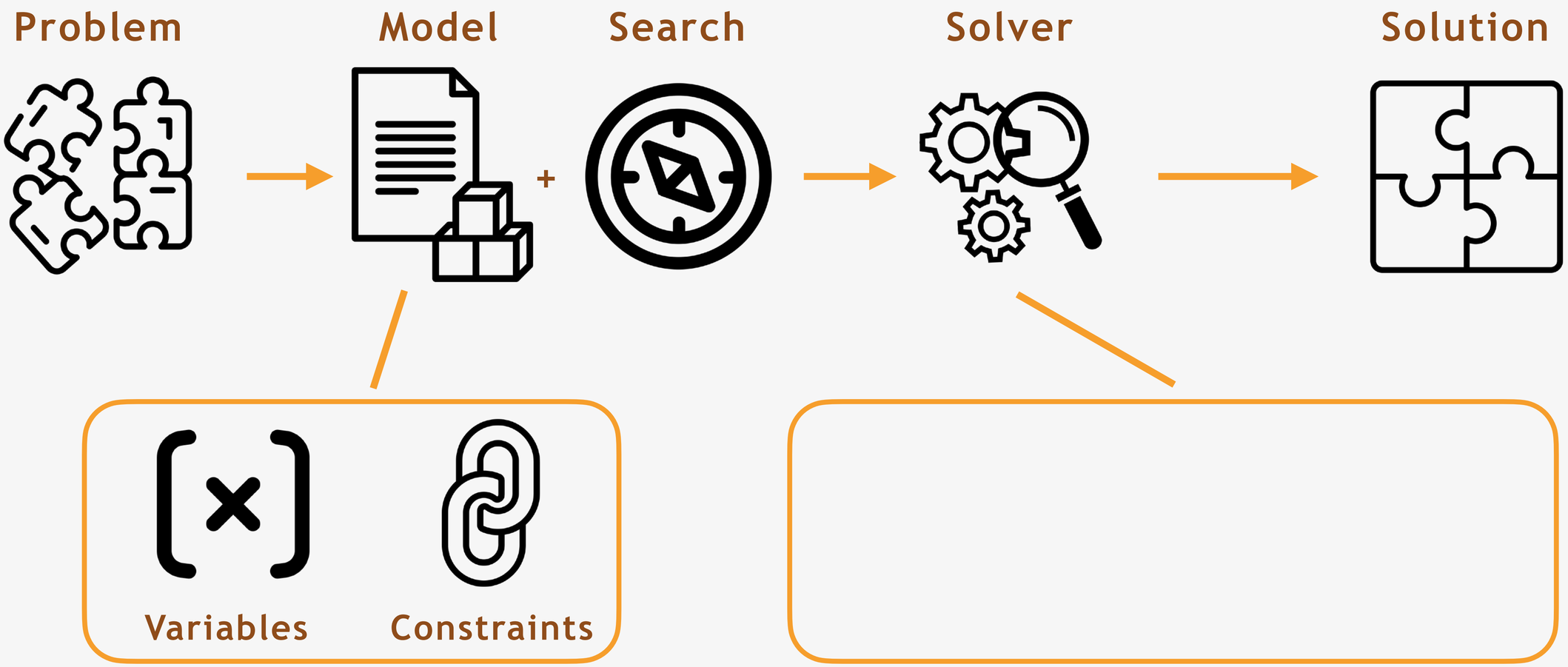

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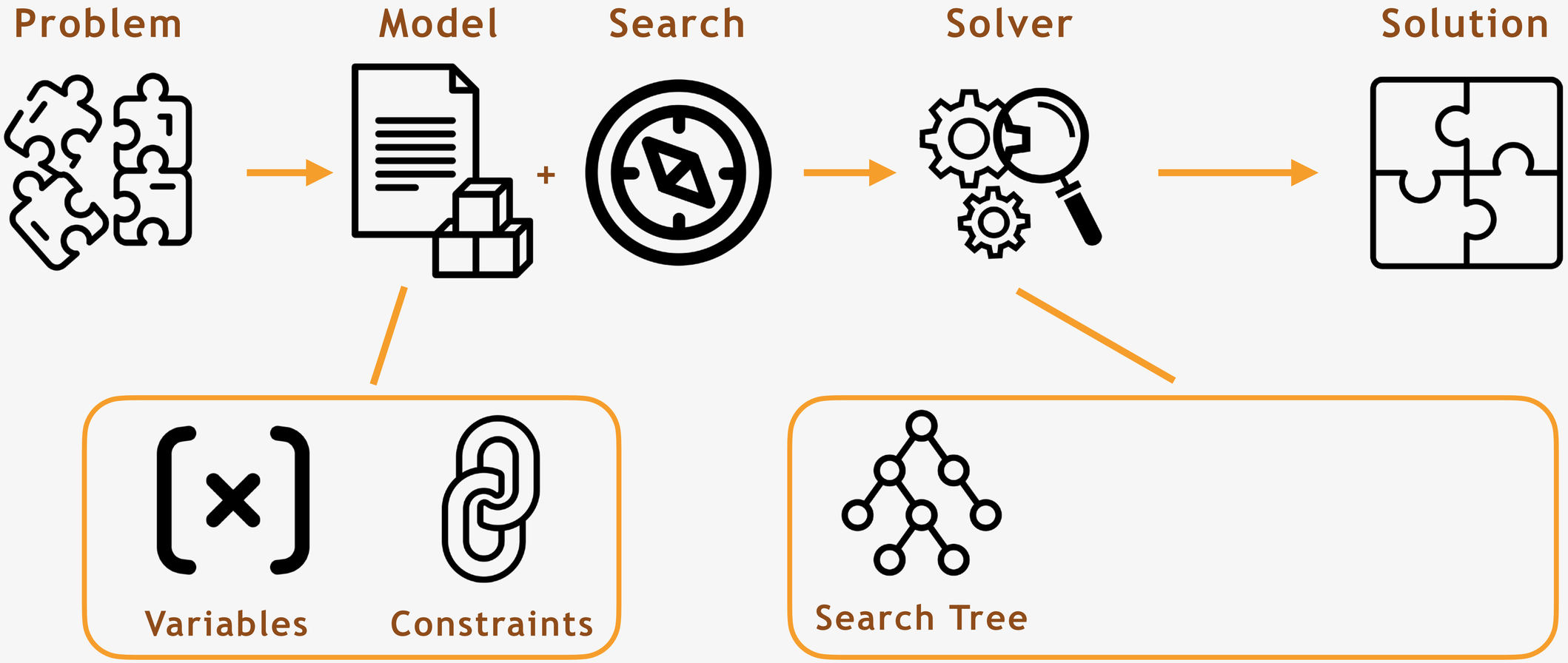

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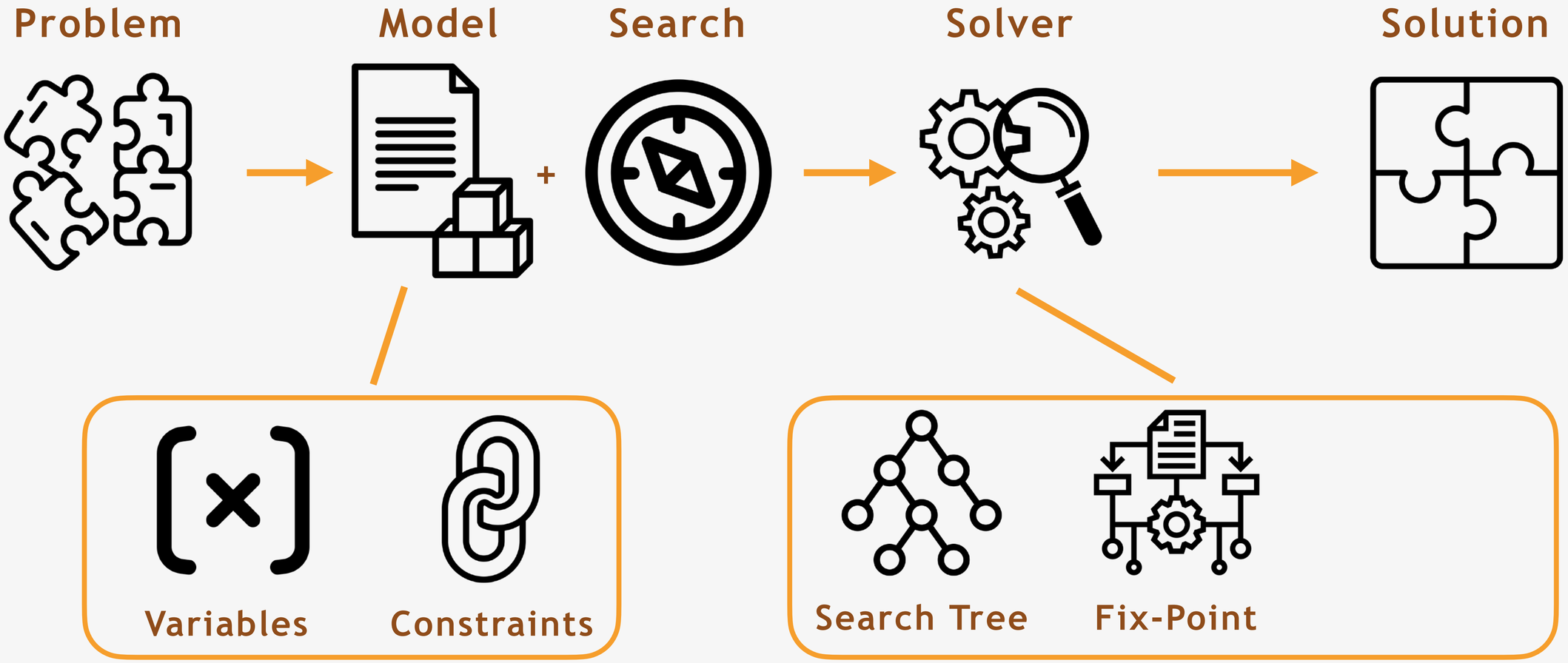

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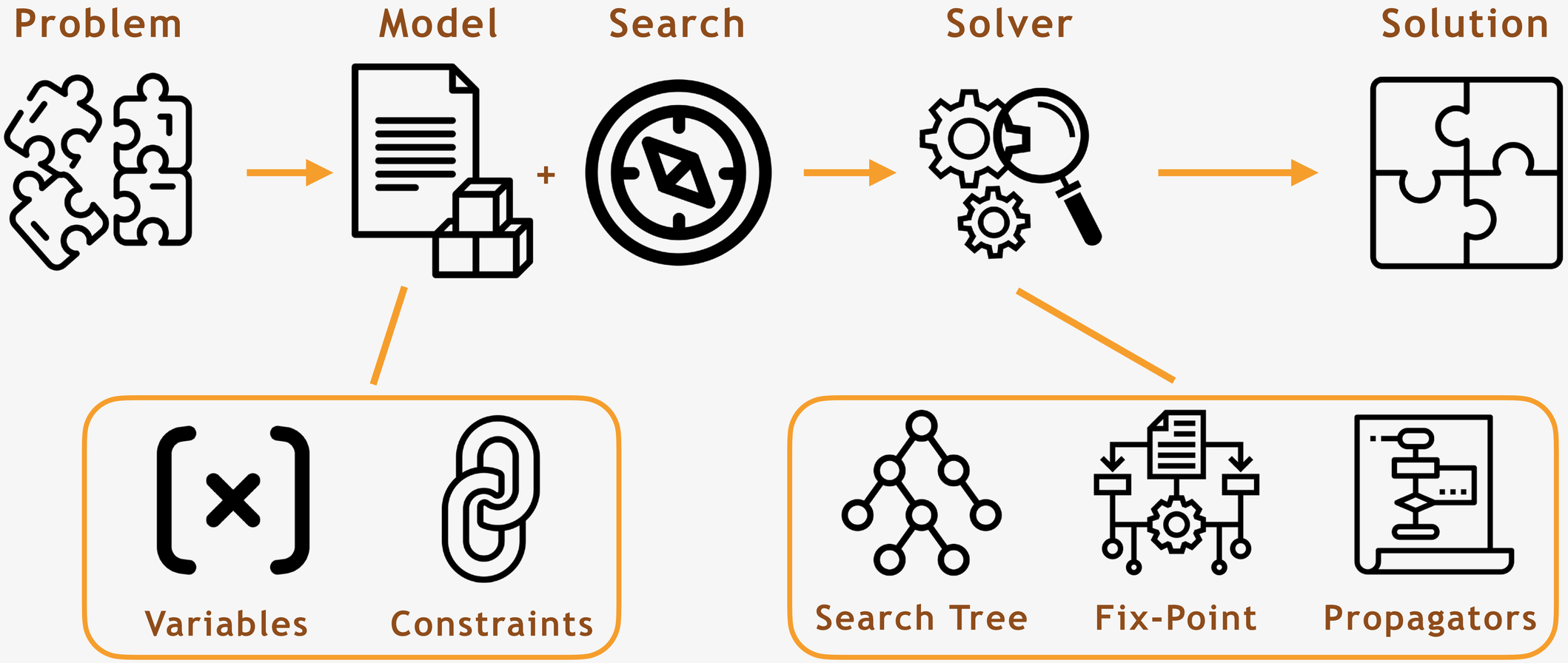

CP = Model + Search

Model

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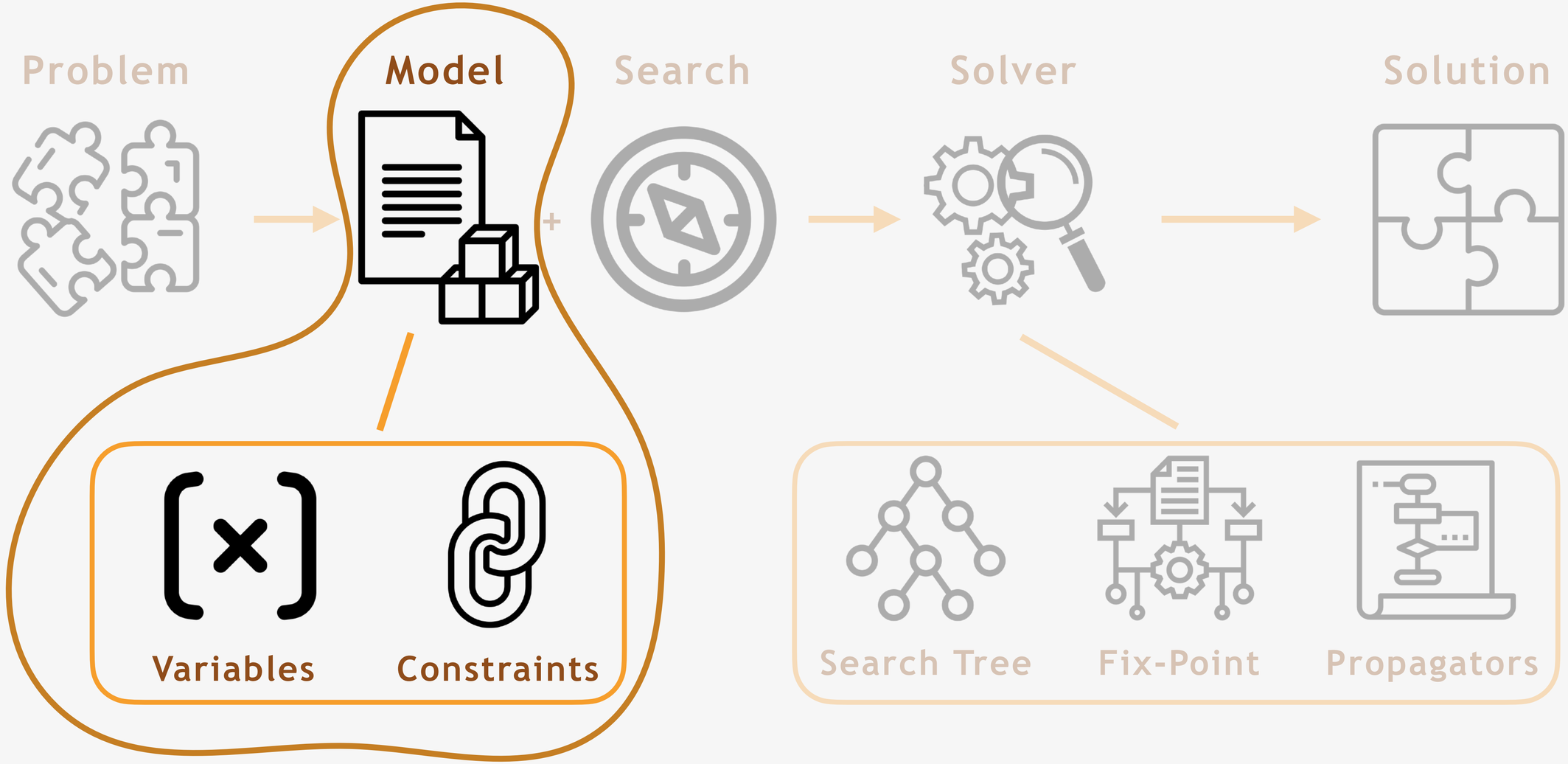

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5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
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- Variables grid[i][j] for i,j ∈ {0..8}
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Let us do the exercise and describe what a solution should be!

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Global Constraint

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Exemple of variables:

- X, with dom(X) = {0,1} is a boolean variable
- Y, with dom(Y) = {2,4,6} is an integer variable

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- Circuit(X,Y,Z)
- Cumulative(starts, durations, ressources)

MODELLING IN MORE DETAILS

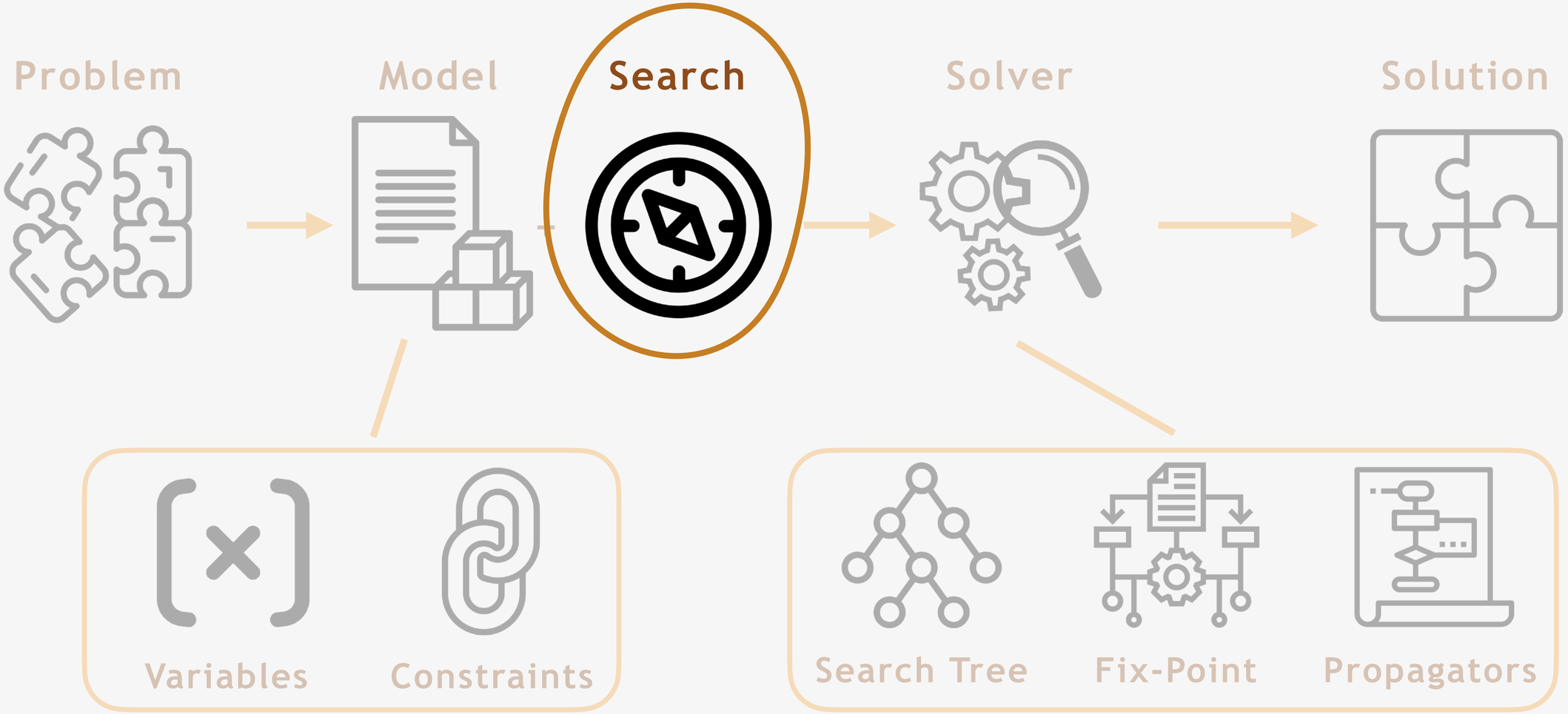
-Monday 11:00-12:30: Lecture

-Monday 14:00-15:30: Lab

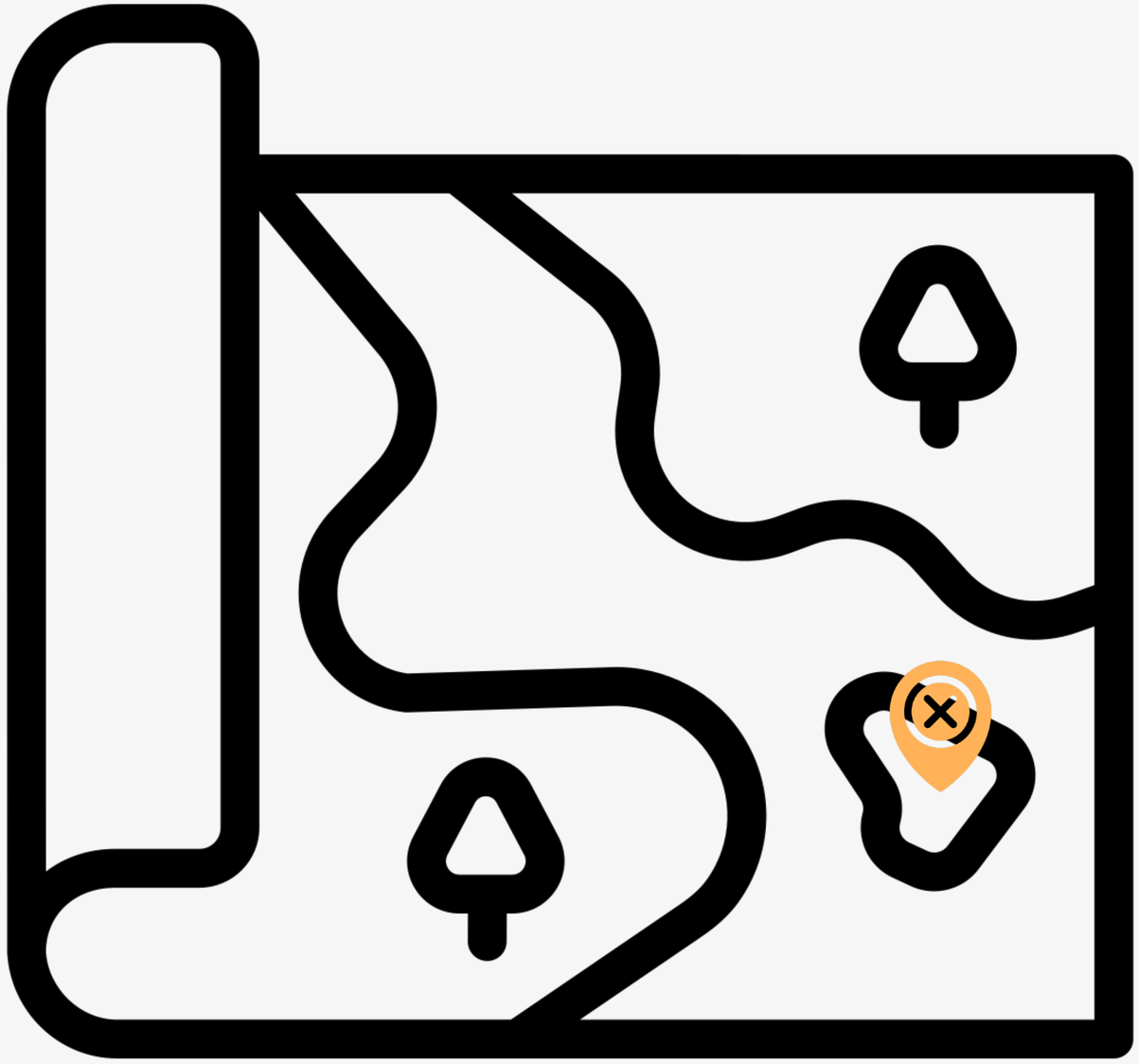
-Monday 16:00-17:30: Lab

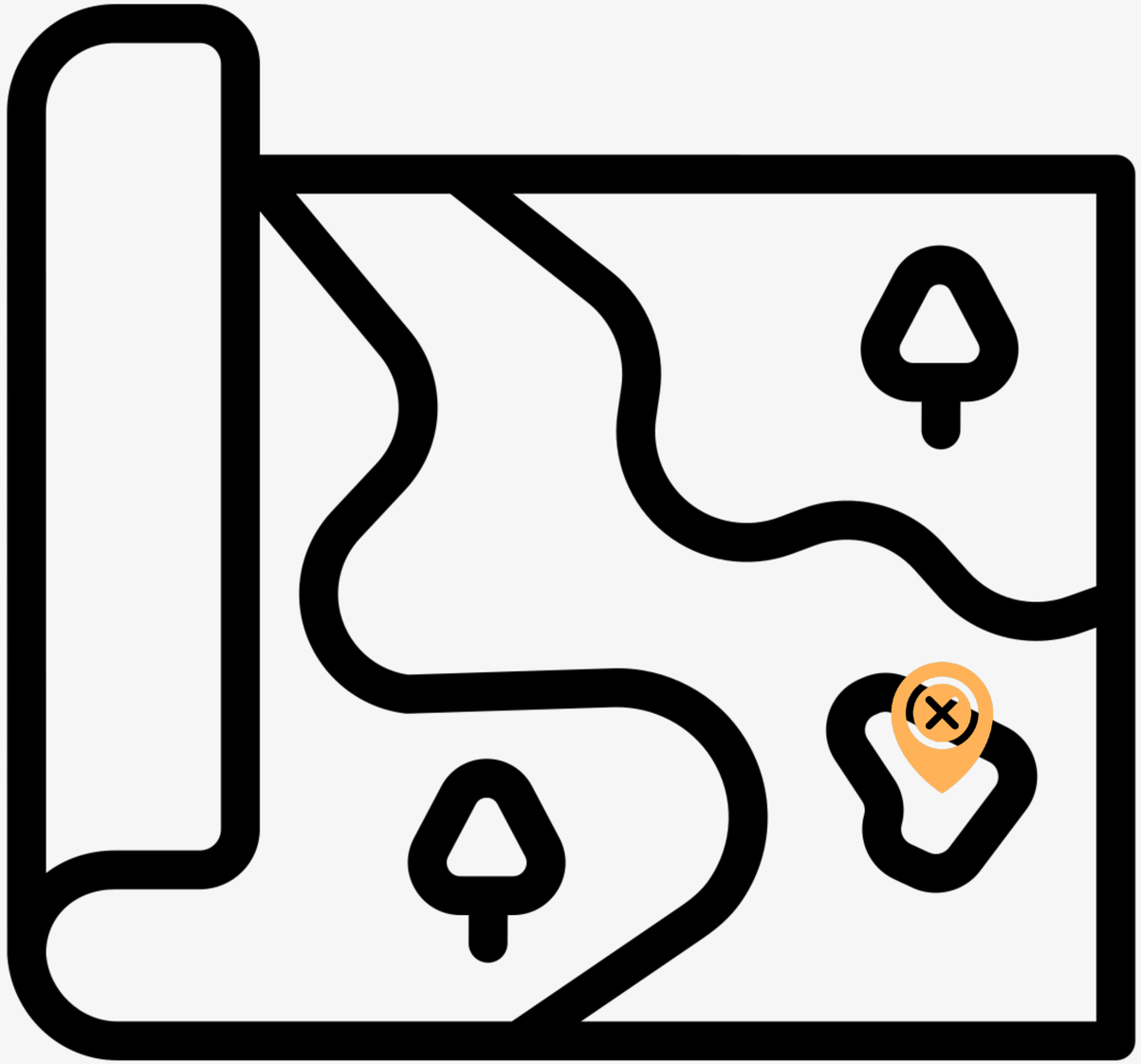
Search

THE CONSTRAINT PROGRAMMING PARADIGM

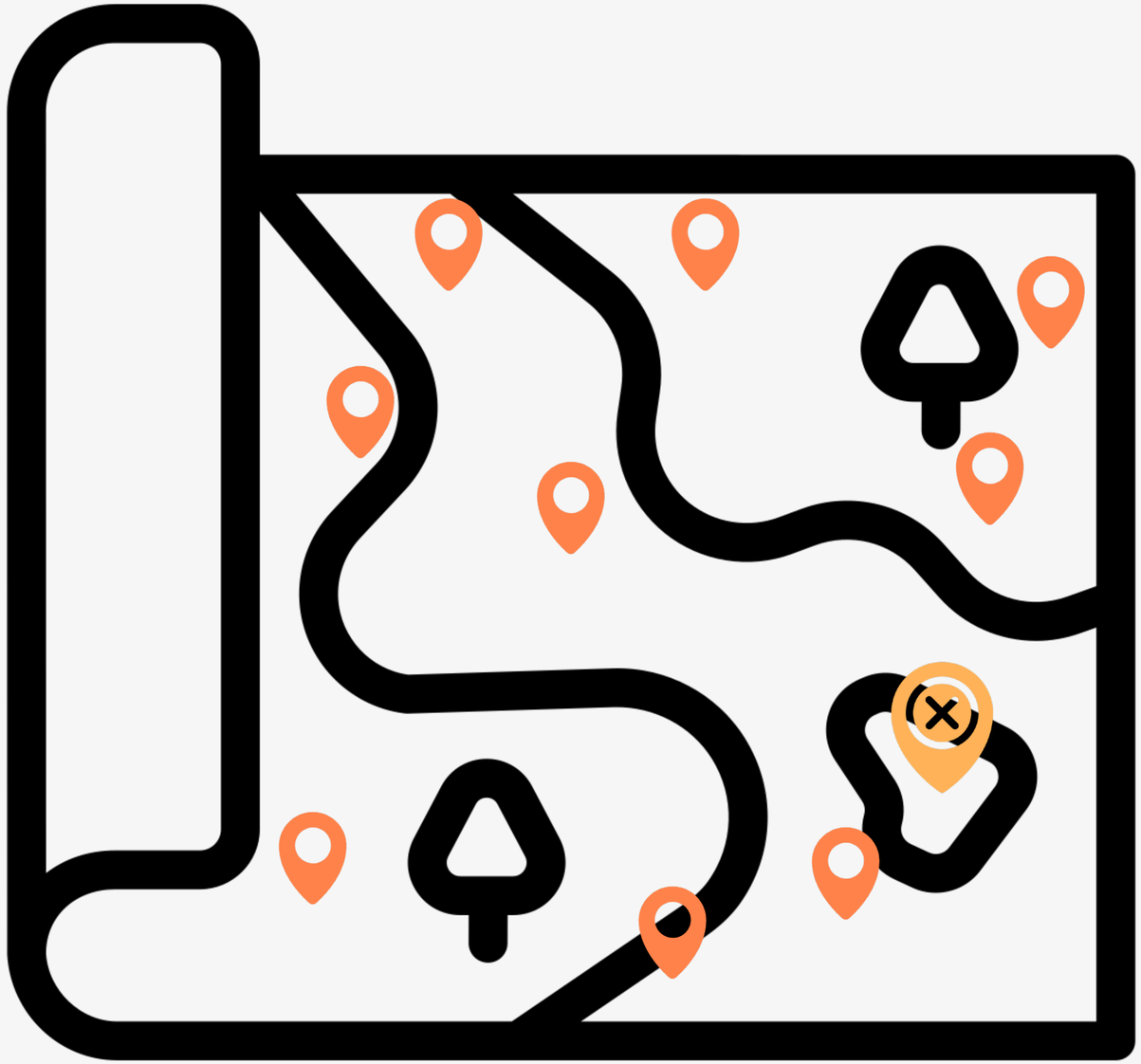


SEARCH





If you know the type of terrain (i.e., the type of problem), then you can have an idea on how to explore it!



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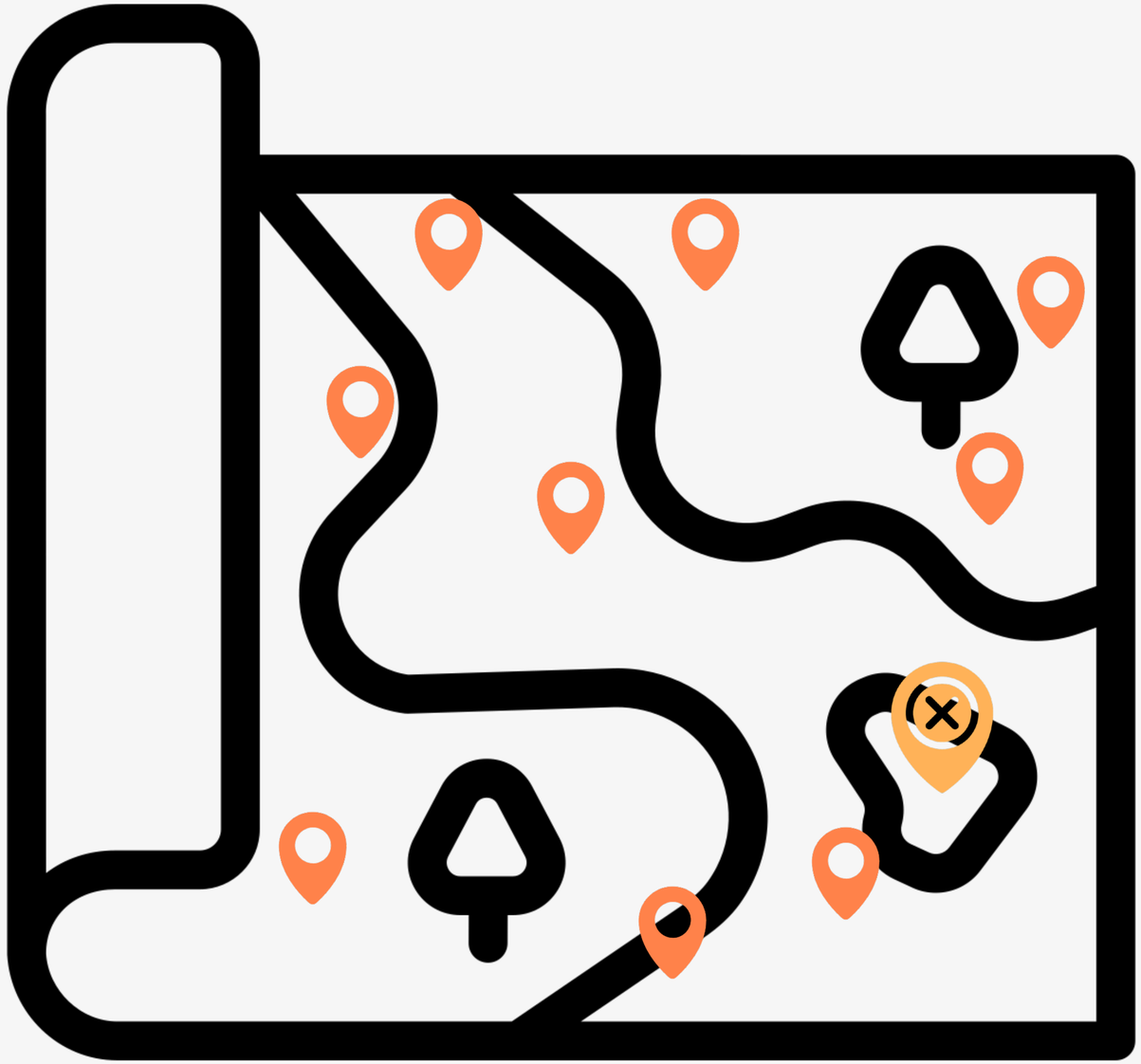

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The search given to the solver is guide you select. The choice is often made based on the knowledge you have of the problem

SEARCH IN MORE DETAILS

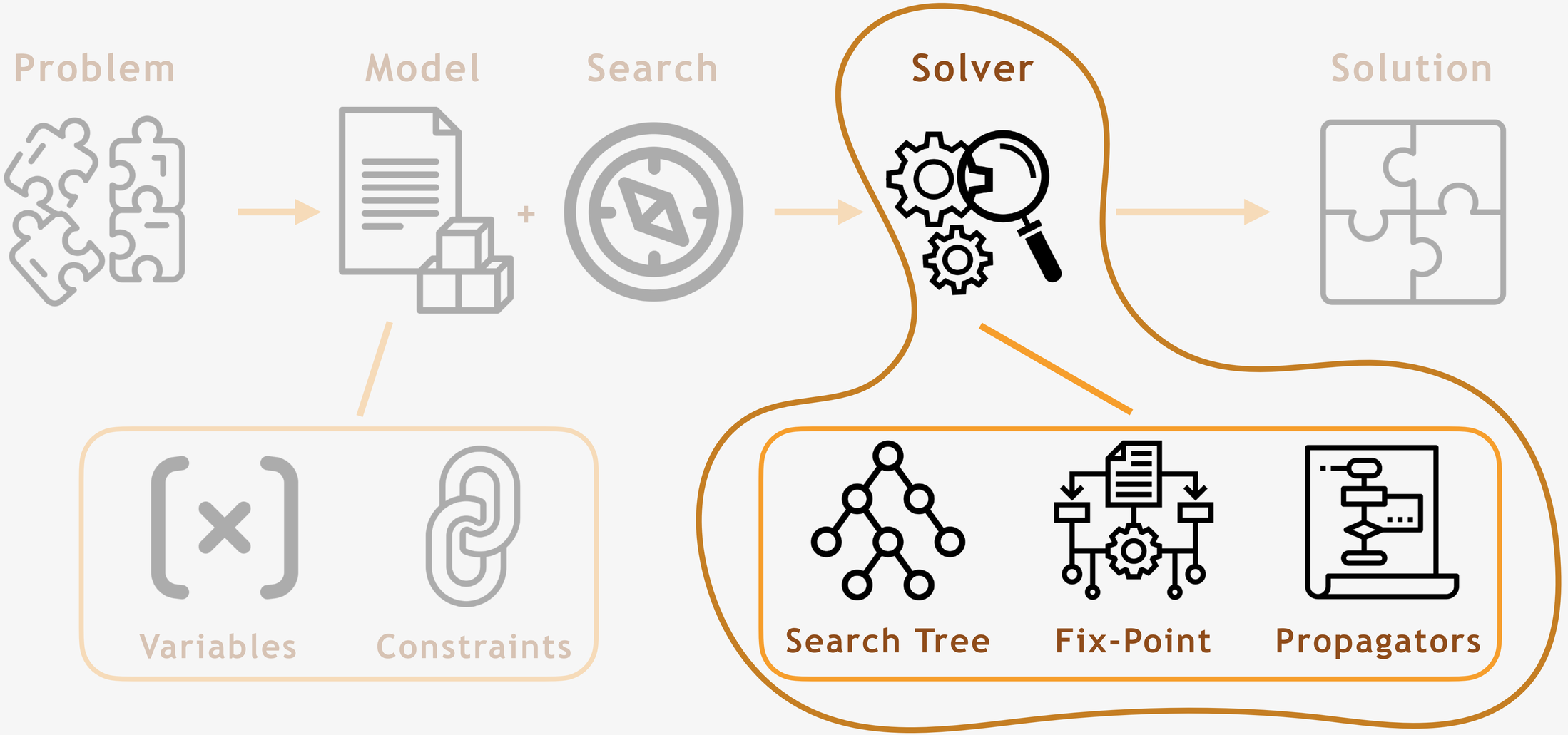
-Wednesday 9:00-10:30: Lecture

-Wednesday 11:00-12:30: Lecture

-Thursday 14:00-15:30: Lab

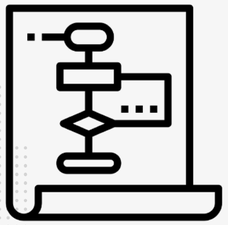
The Solver

THE CONSTRAINT PROGRAMMING PARADIGM



Propagators

GOAL OF A PROPAGATOR



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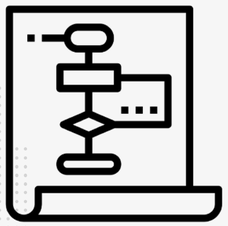

Goal: specialised algorithms which take the current state (i.e., variables and current domains) and filter invalid values using some reasoning corresponding to the constraint

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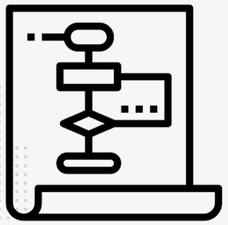

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Example:

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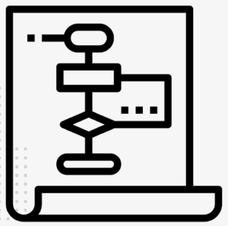

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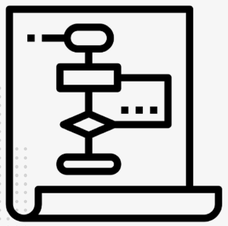

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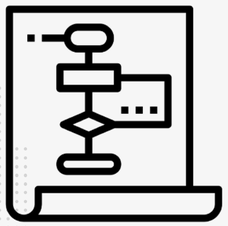

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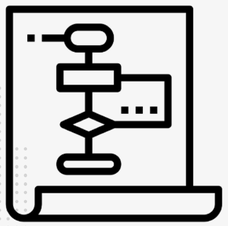

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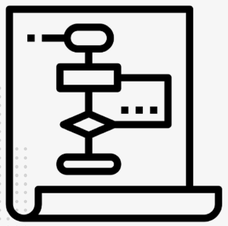
GOAL OF A PROPAGATOR



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PROPAGATORS IN MORE DETAILS

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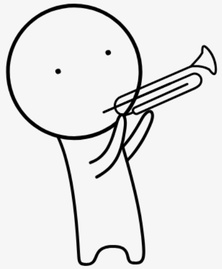
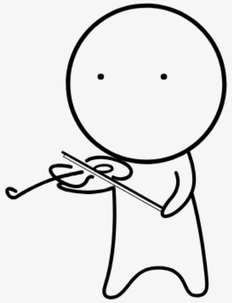
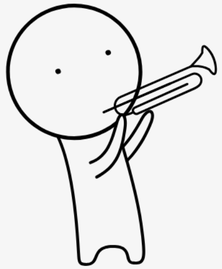
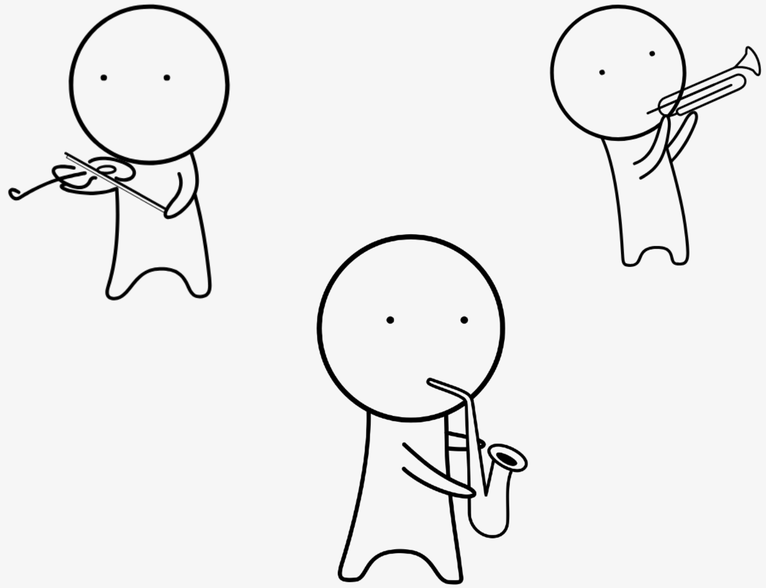
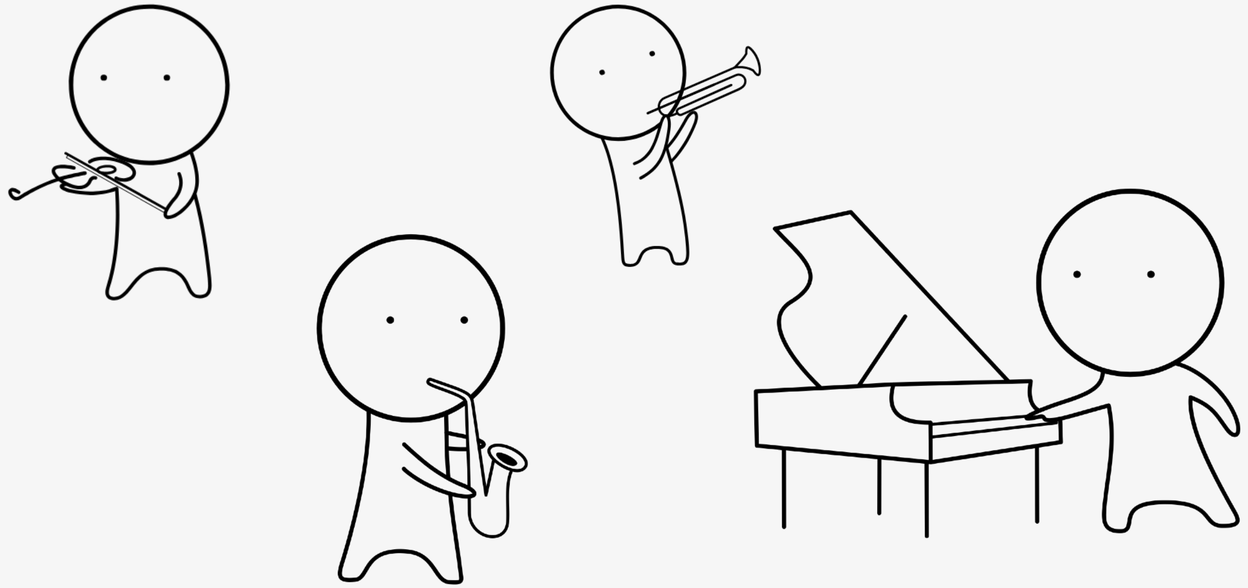
Fixpoint Algorithm

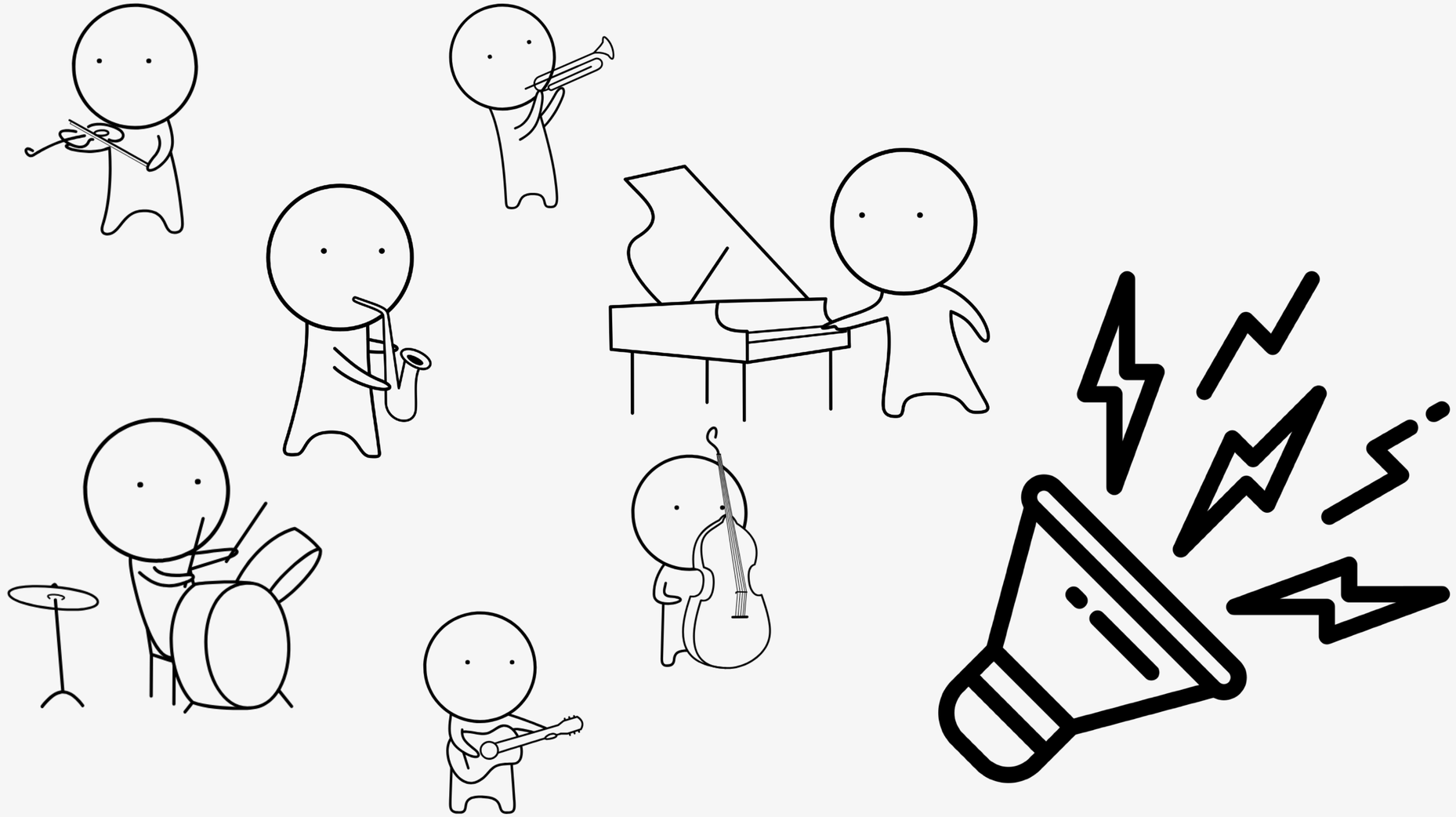


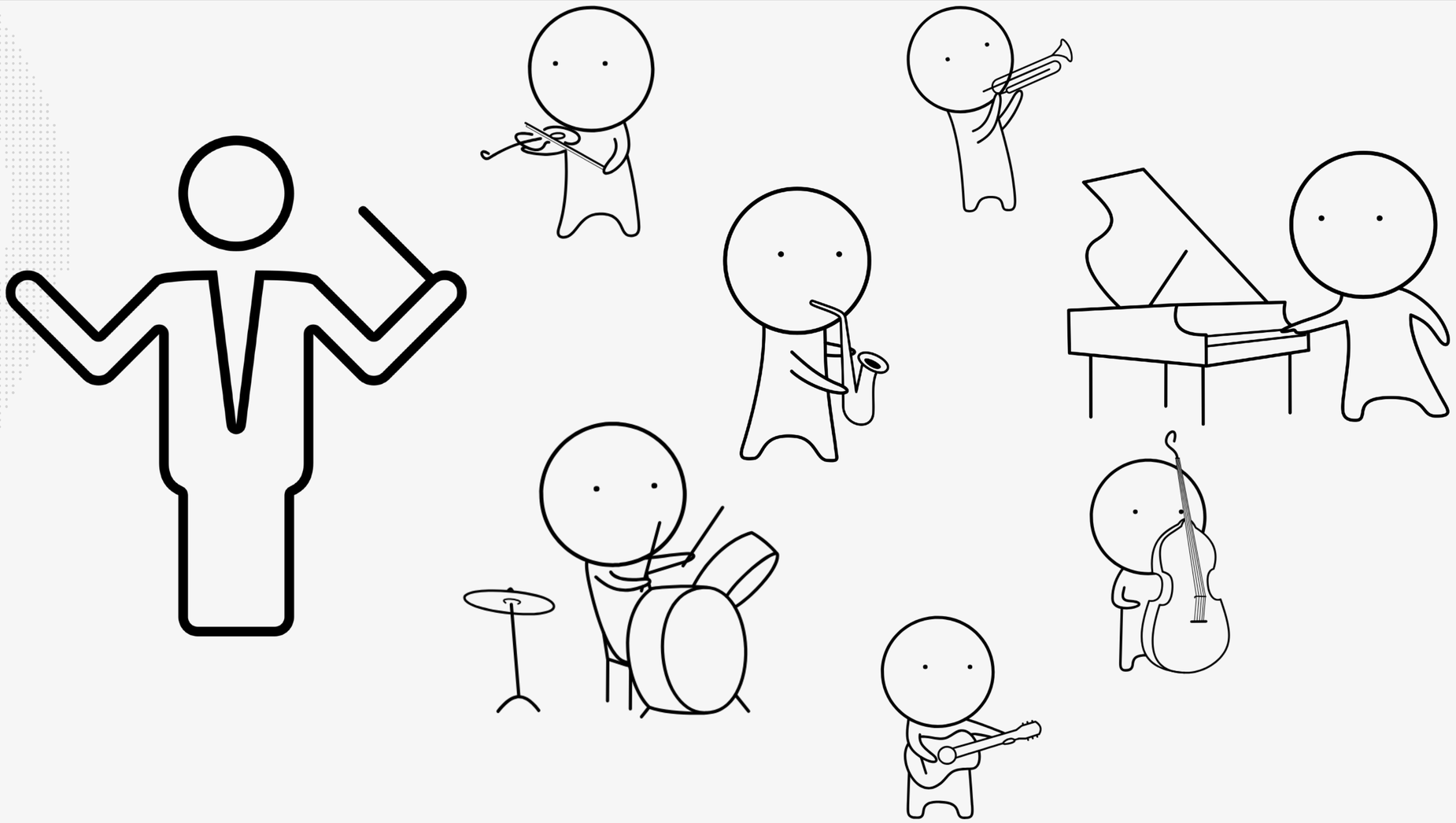


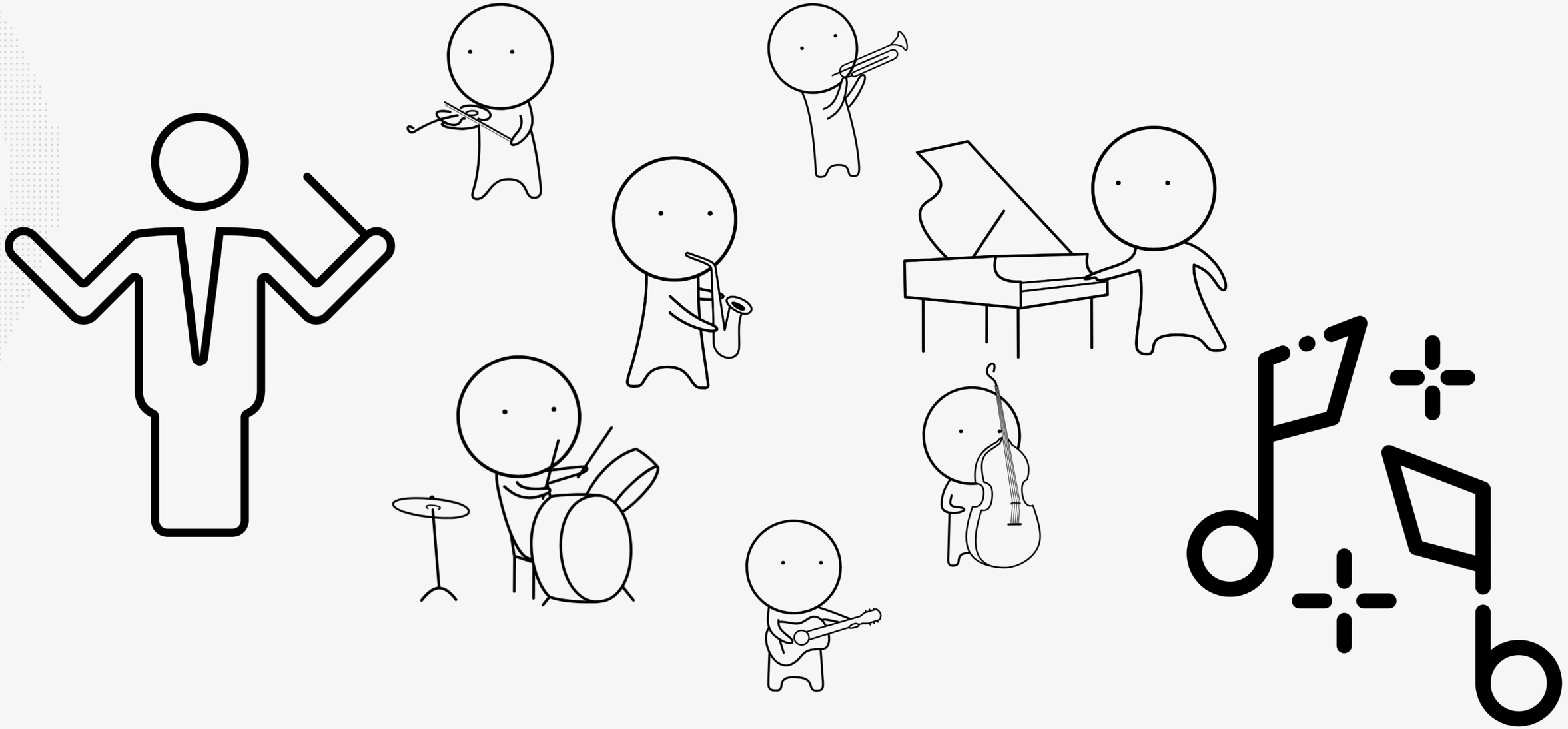




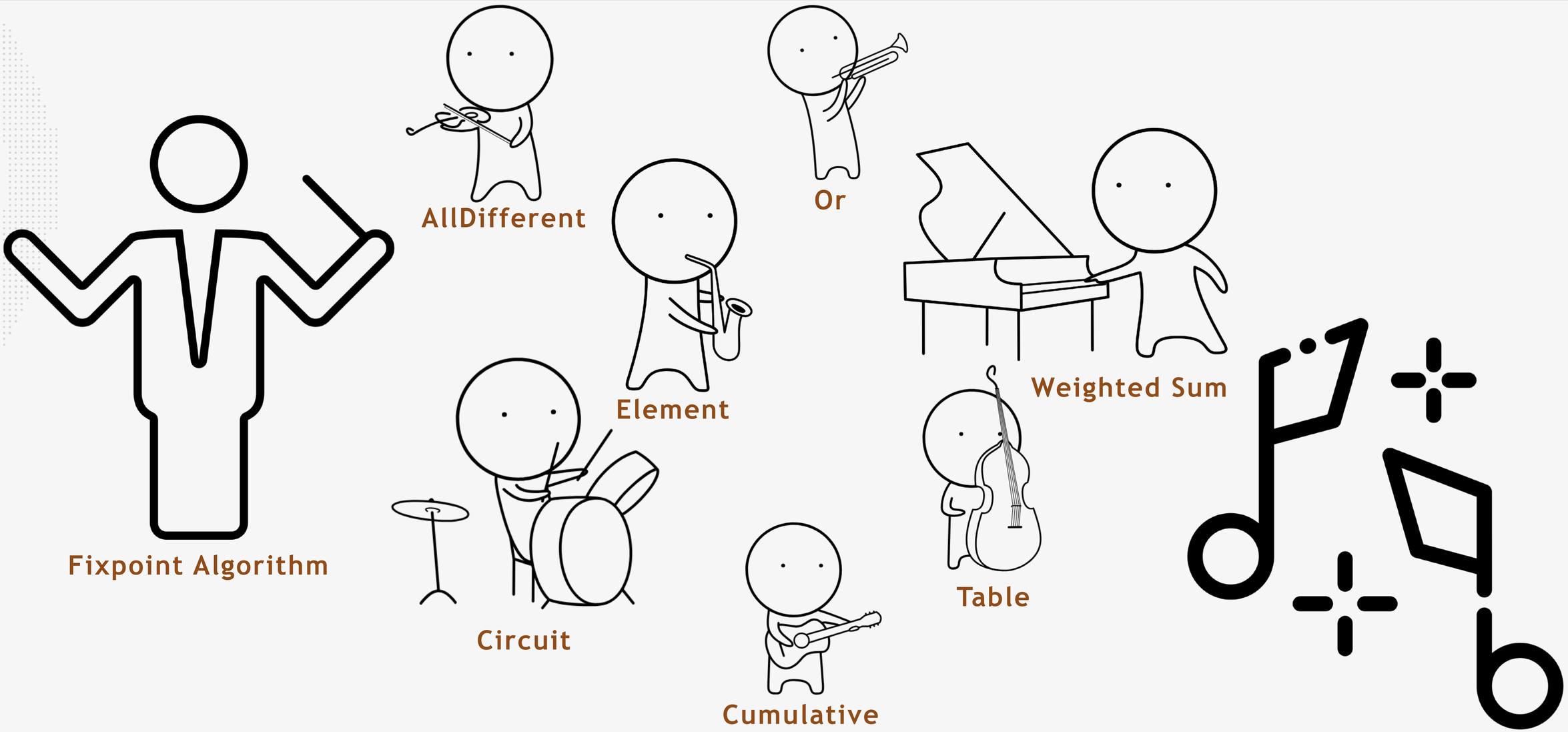






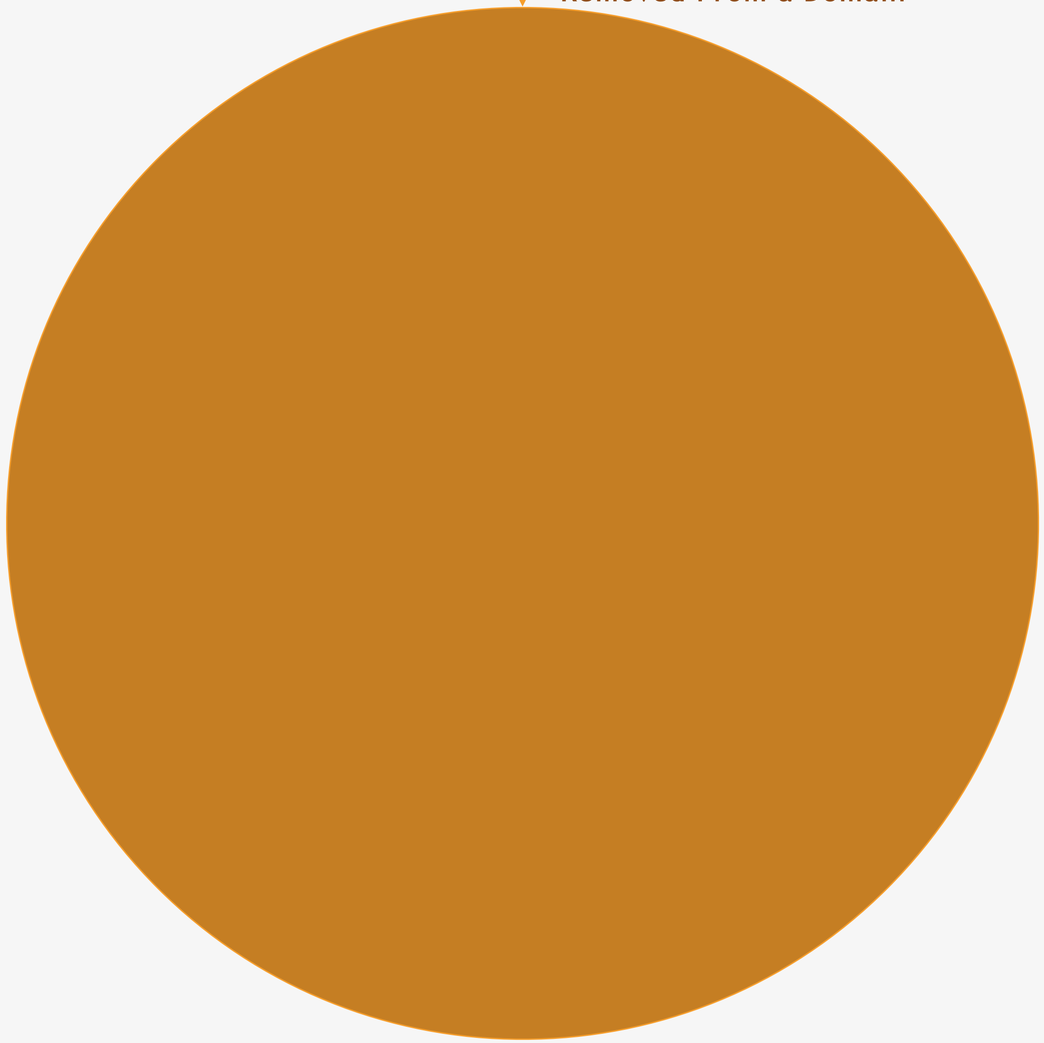


MAESTRO OF THE CONSTRAINTS PROPAGATORS



REACHING A STABLE STATE AGAIN

Perturbation of the State,
I.E., Some Value Is
Removed From a Domain



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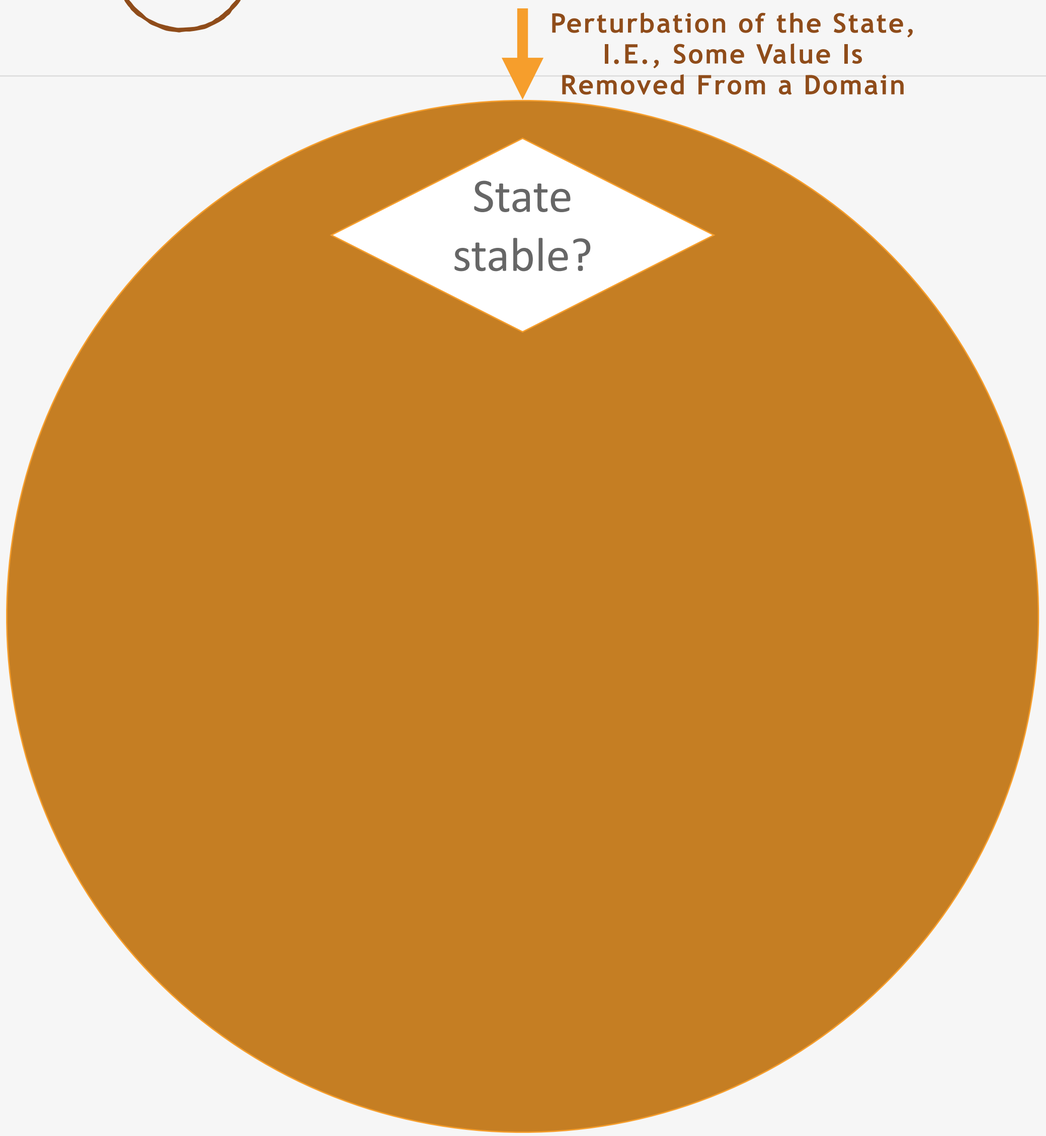

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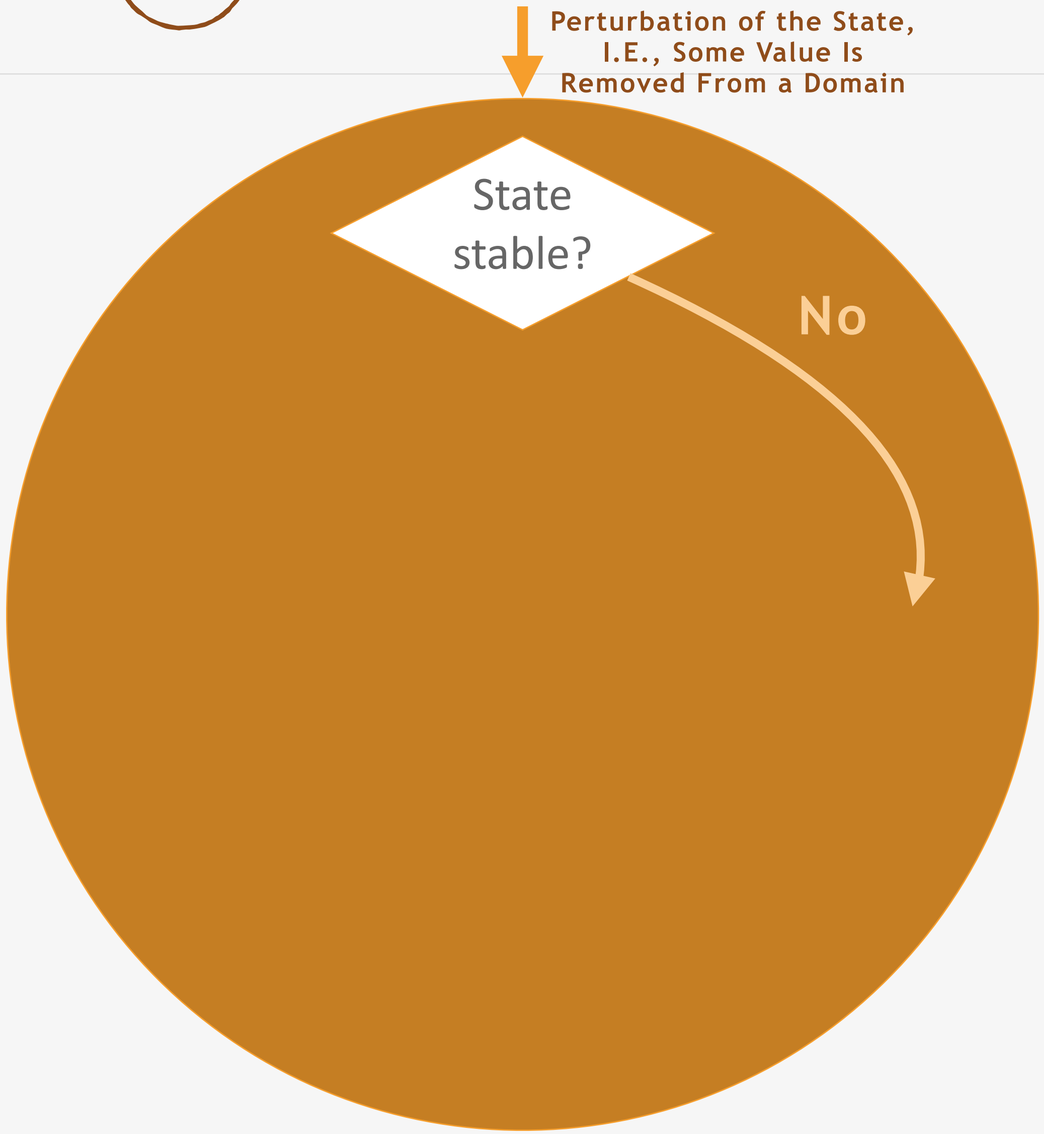
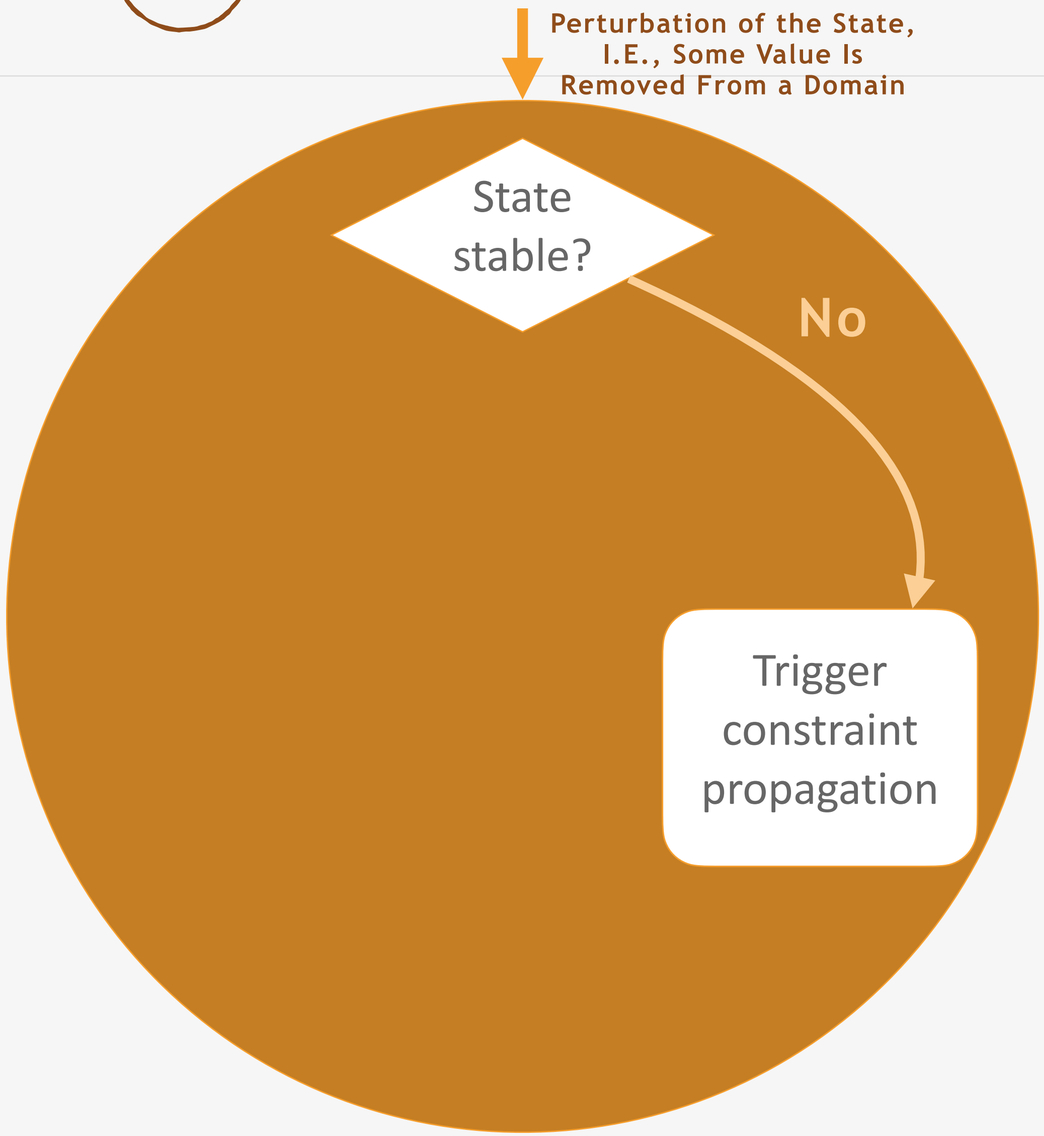


REACHING A STABLE STATE AGAIN



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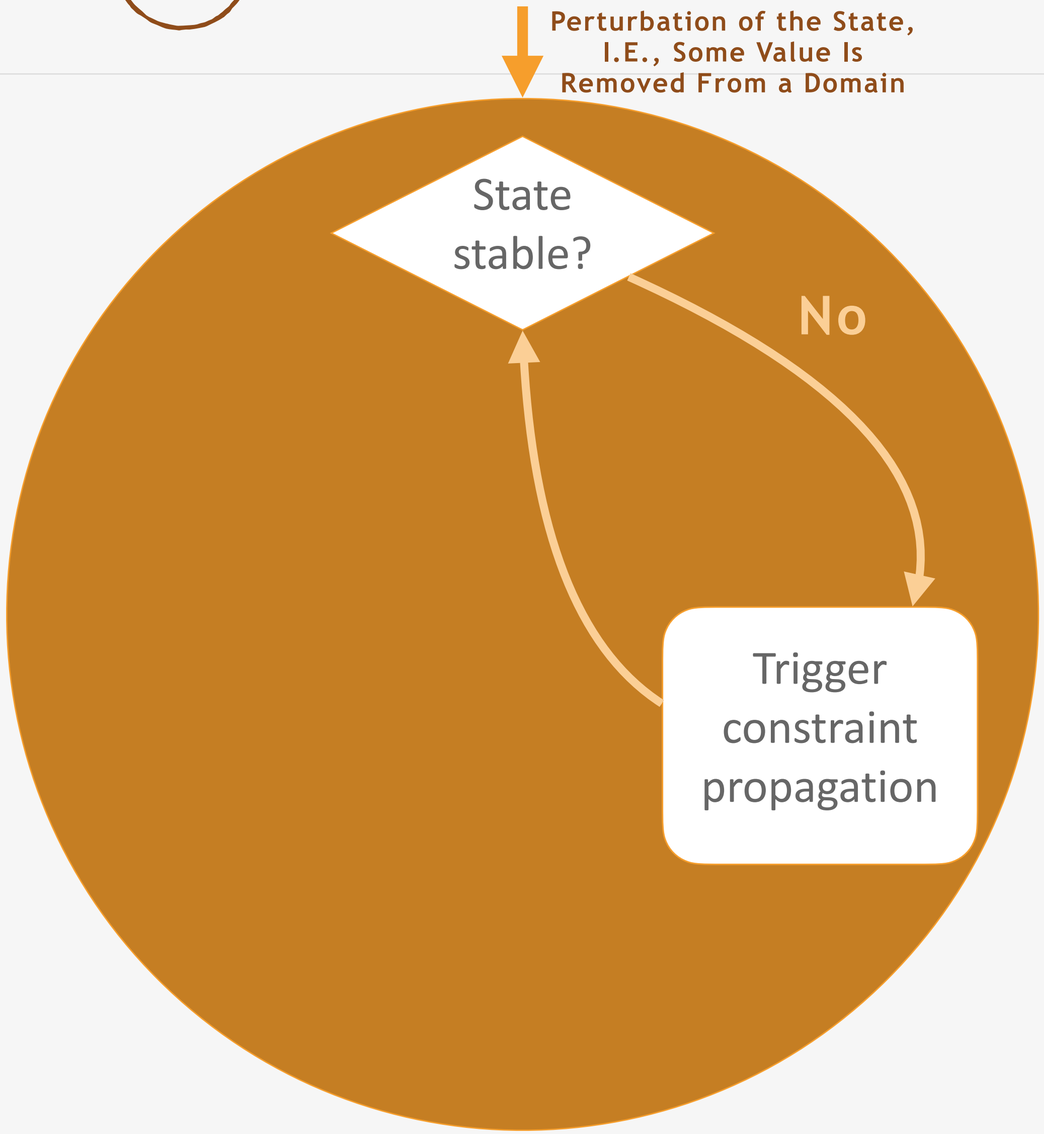

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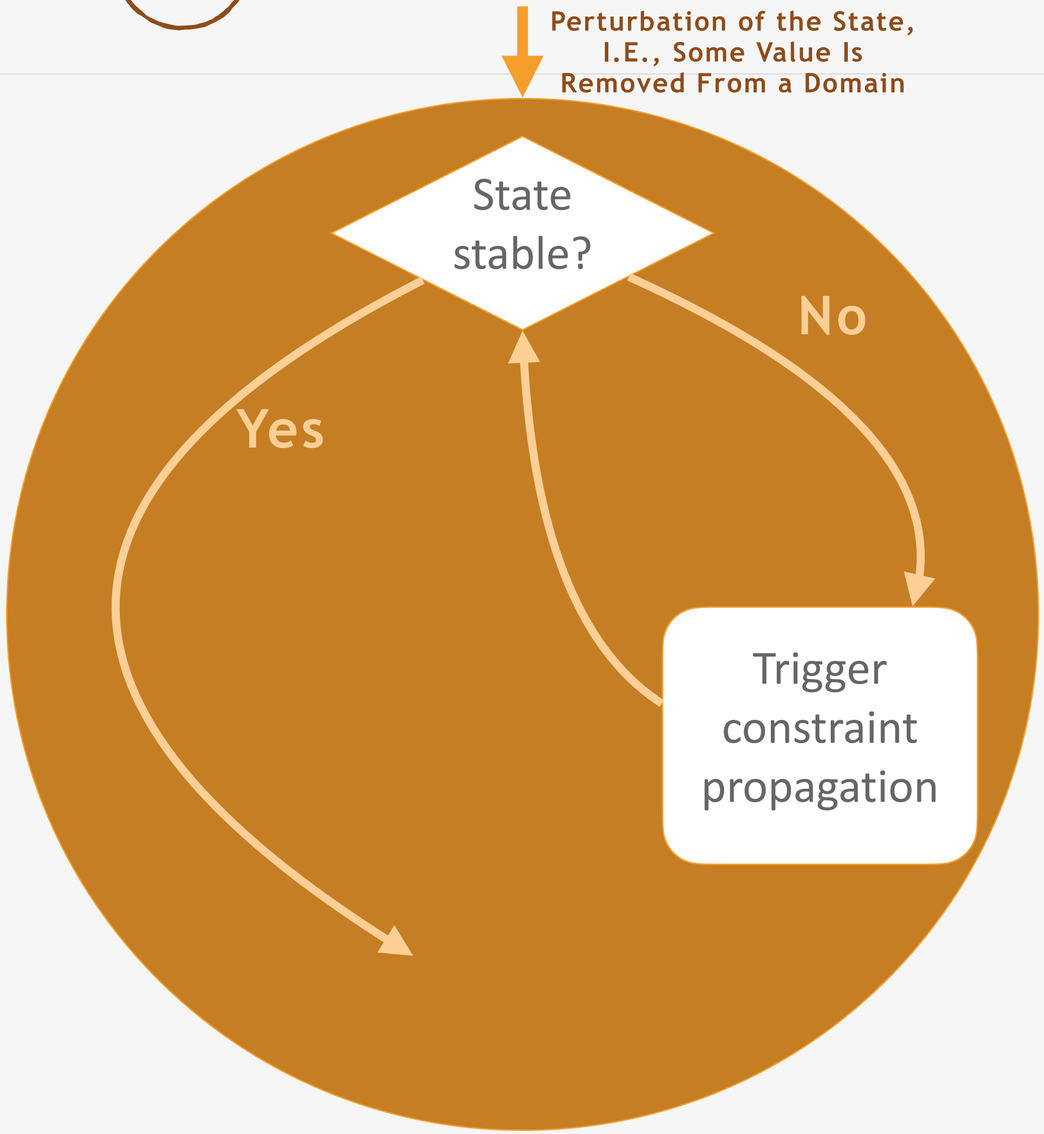

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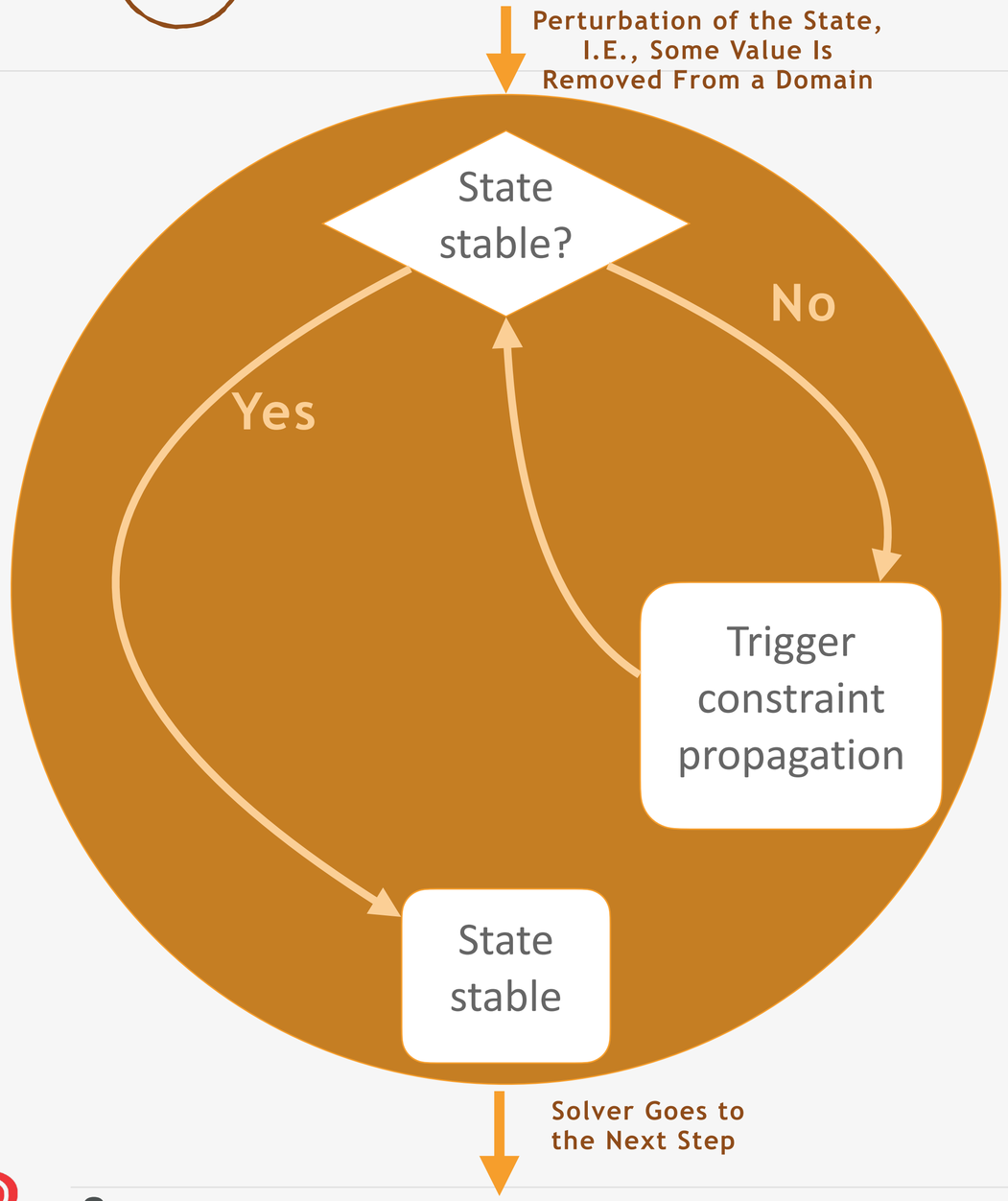

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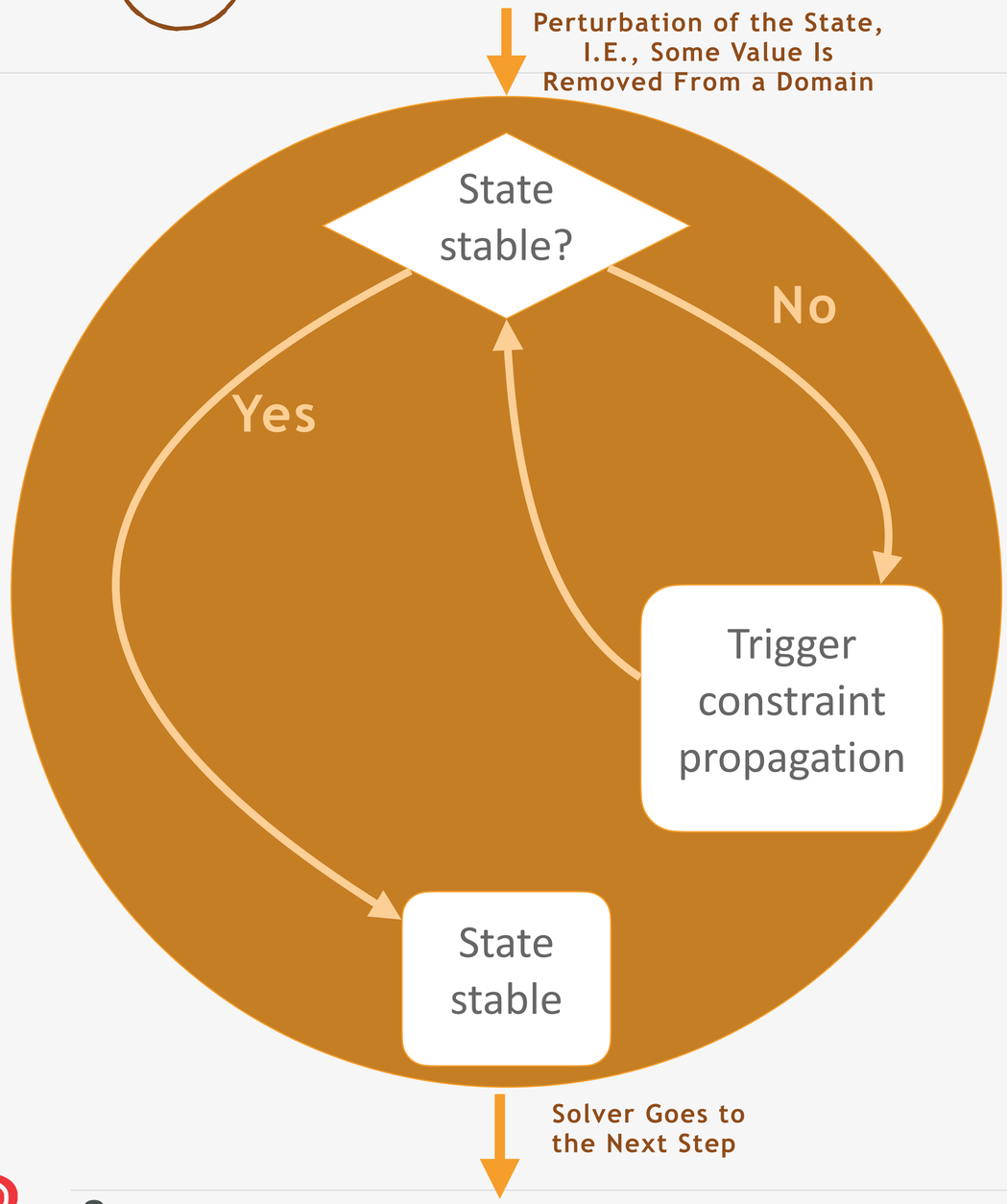
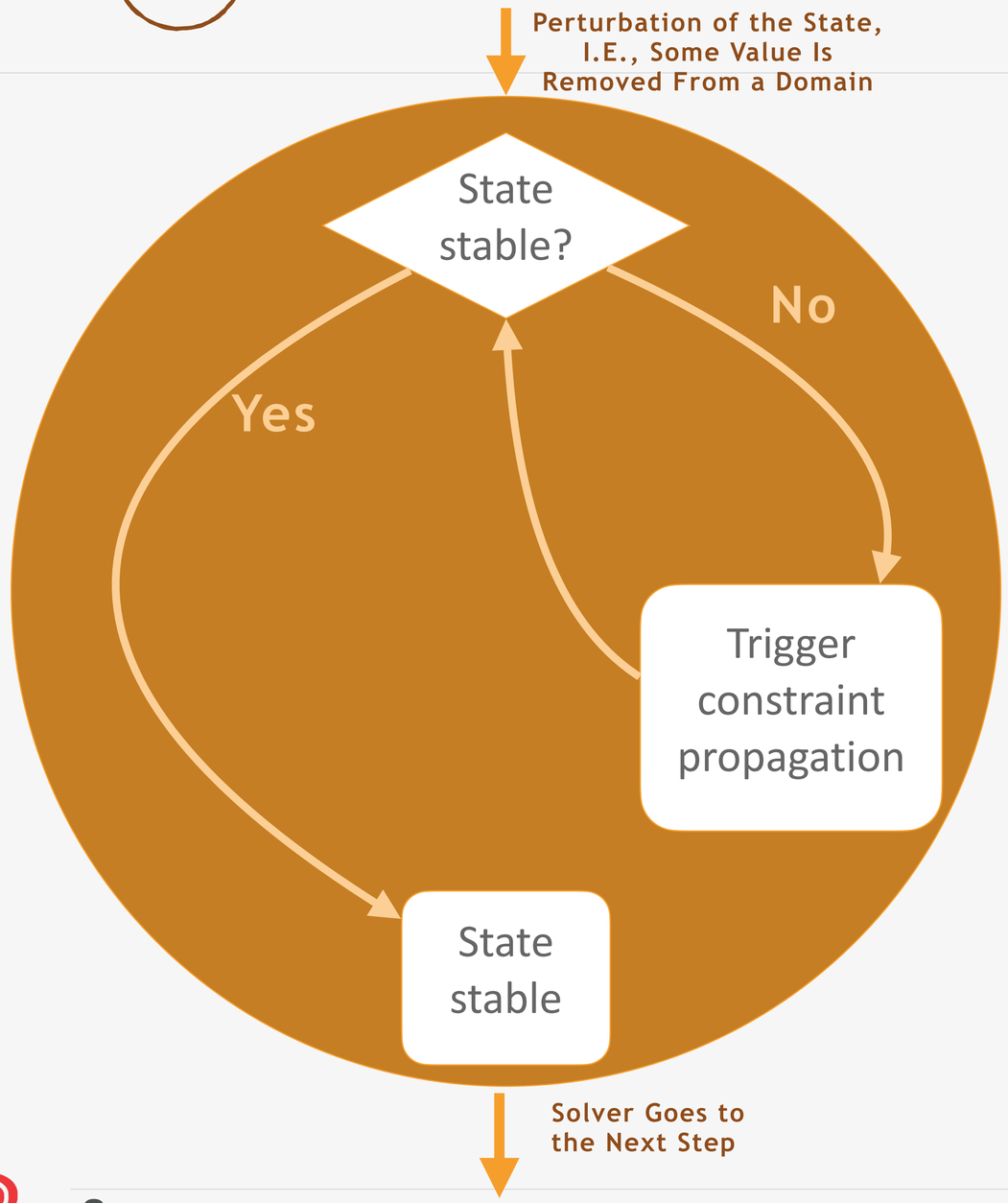


What Is a Stable State?

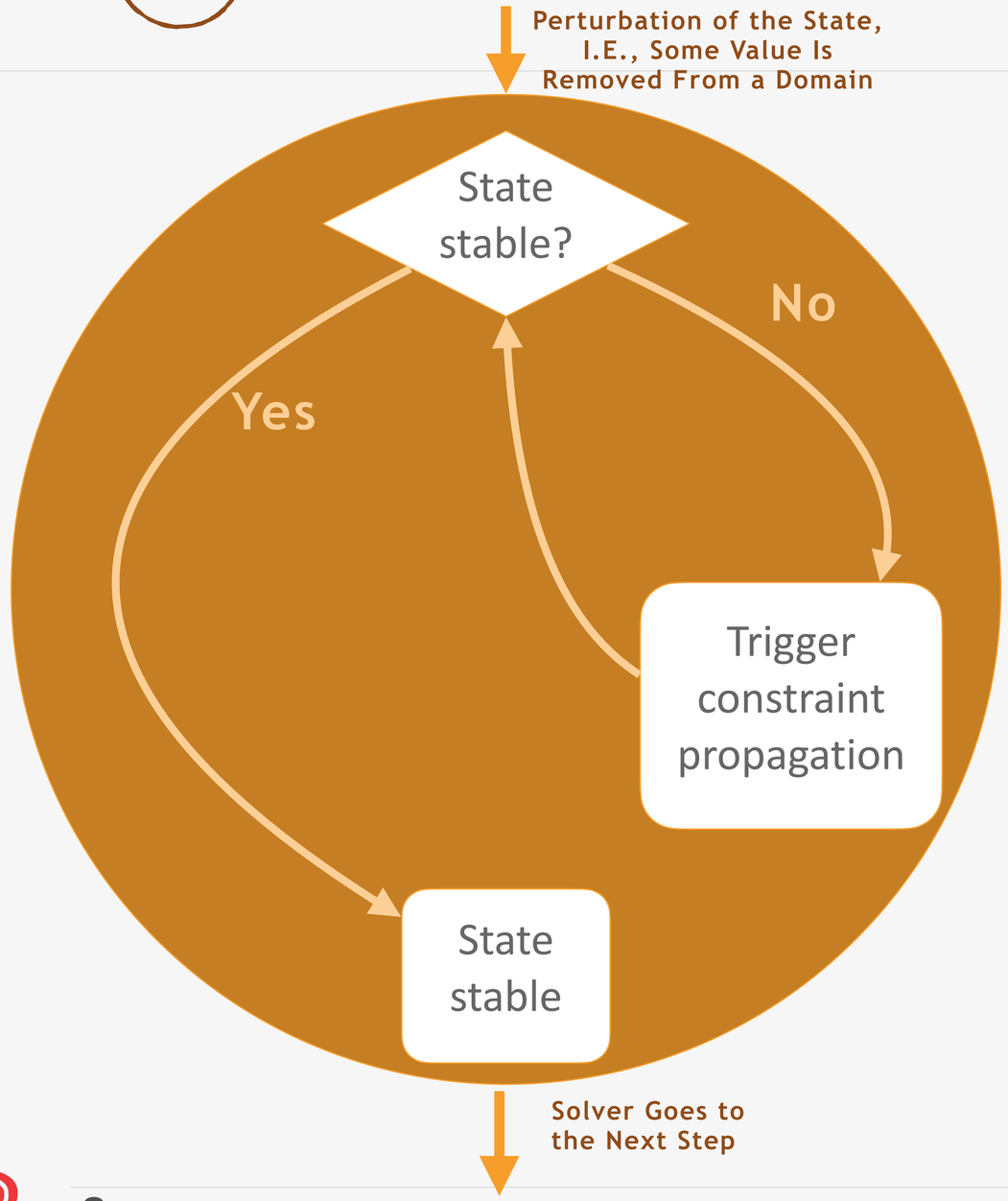
REACHING A STABLE STATE AGAIN



What Is a Stable State?

When no More Constraints Have Been Flagged “To Be Triggered”

REACHING A STABLE STATE AGAIN



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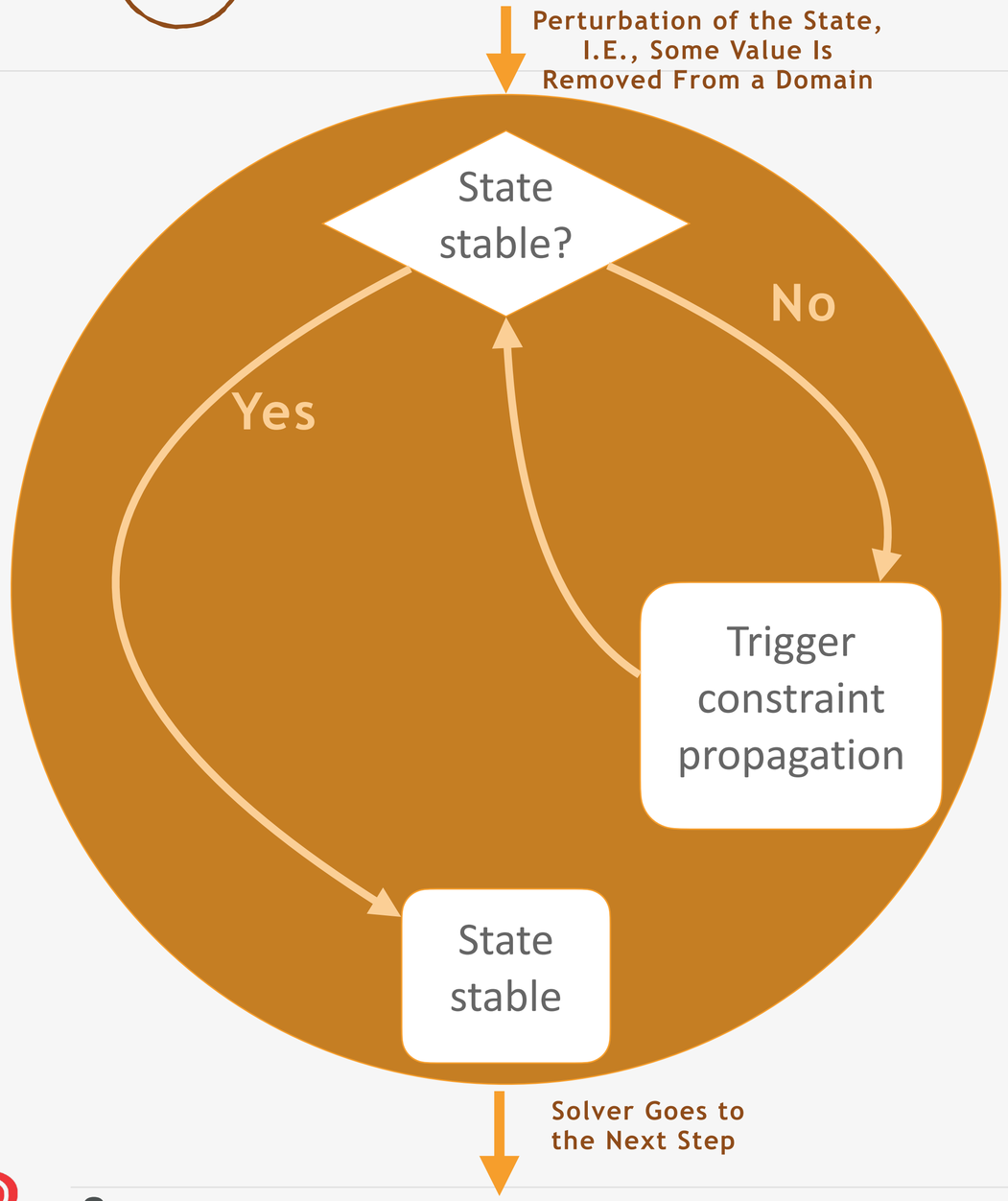
Which Constraint To Trigger Next?

REACHING A STABLE STATE AGAIN



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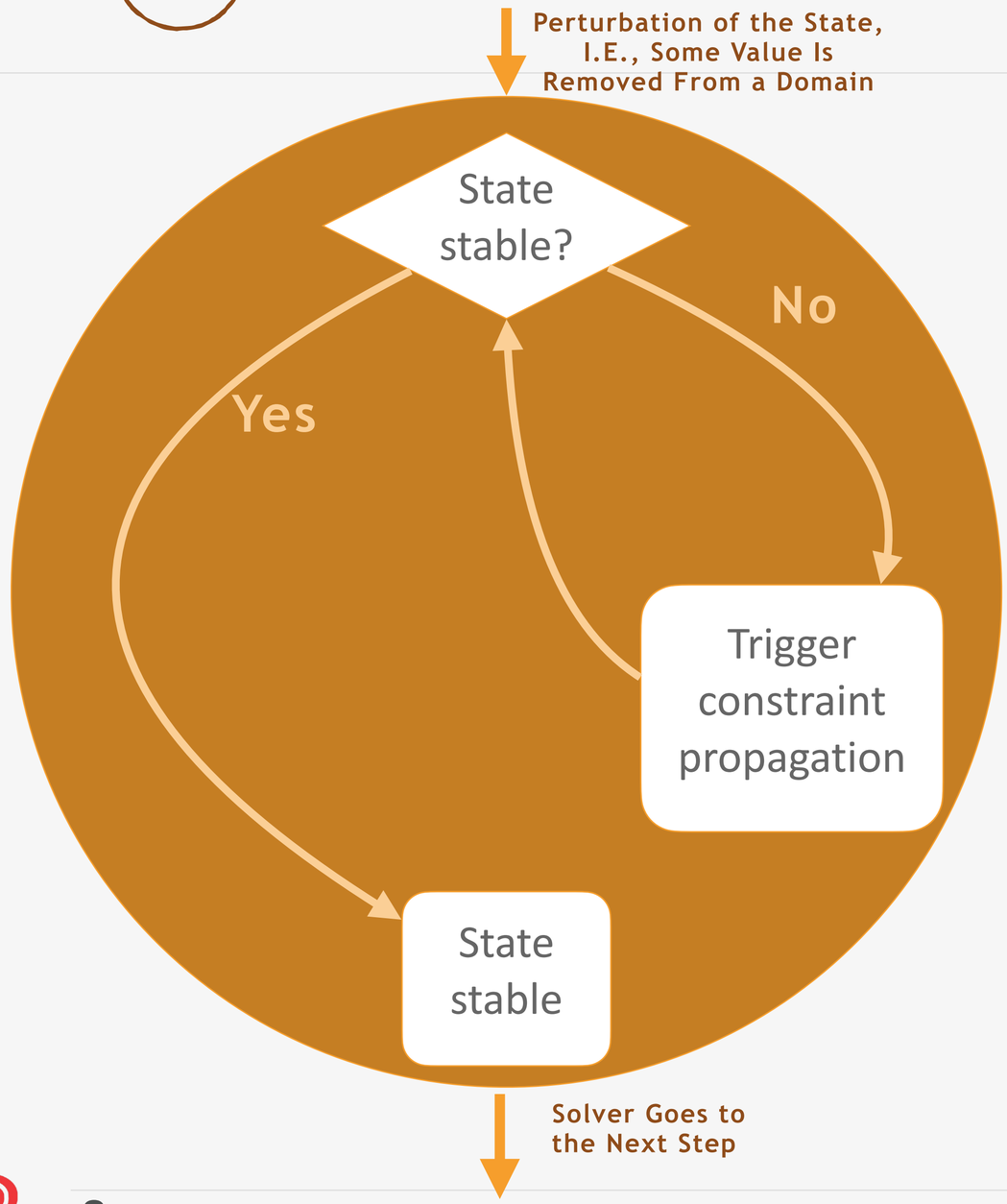

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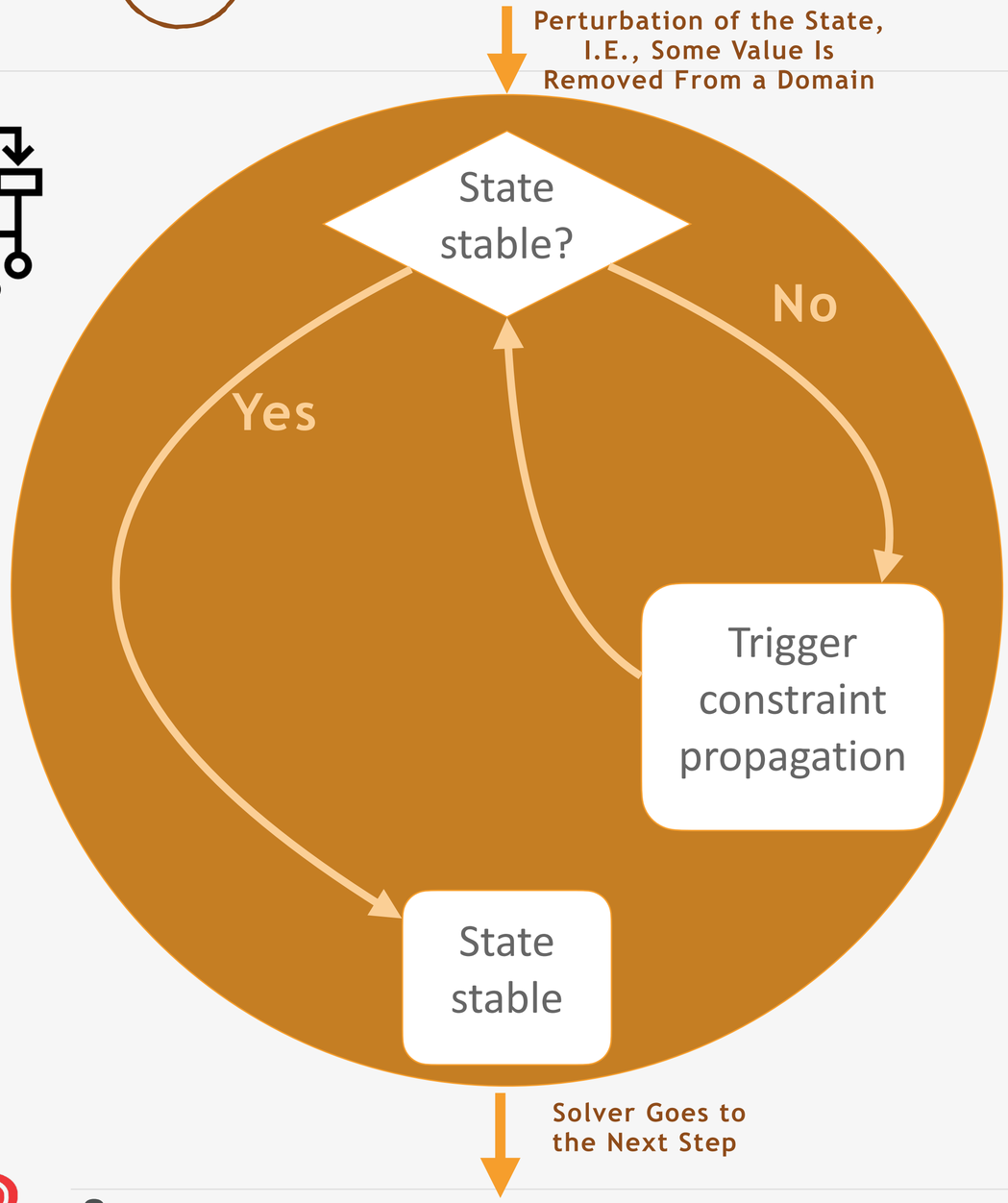

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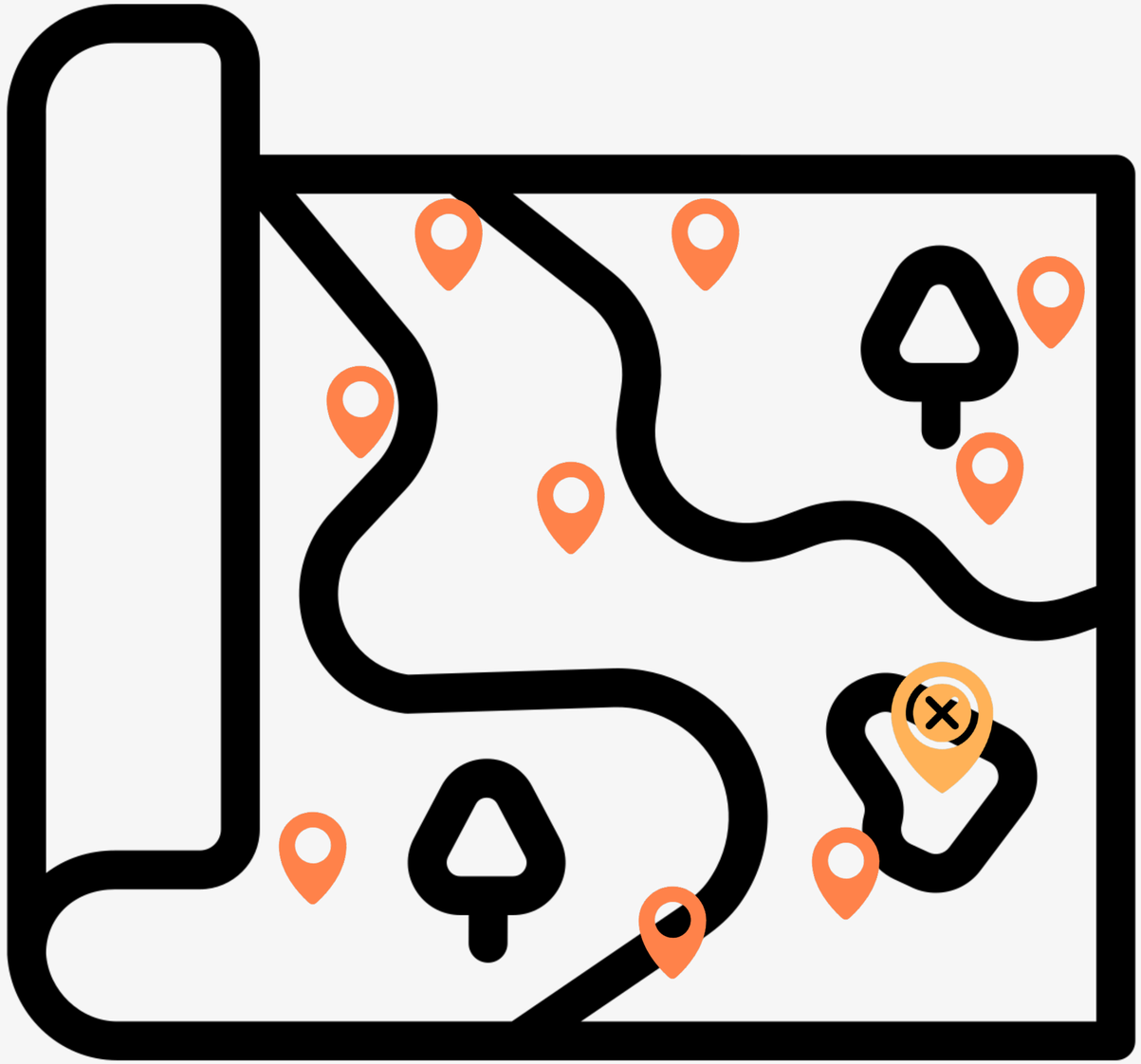
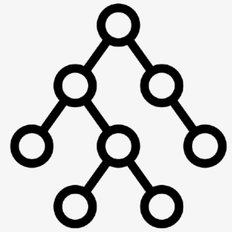
Search Tree

SEARCH TREE





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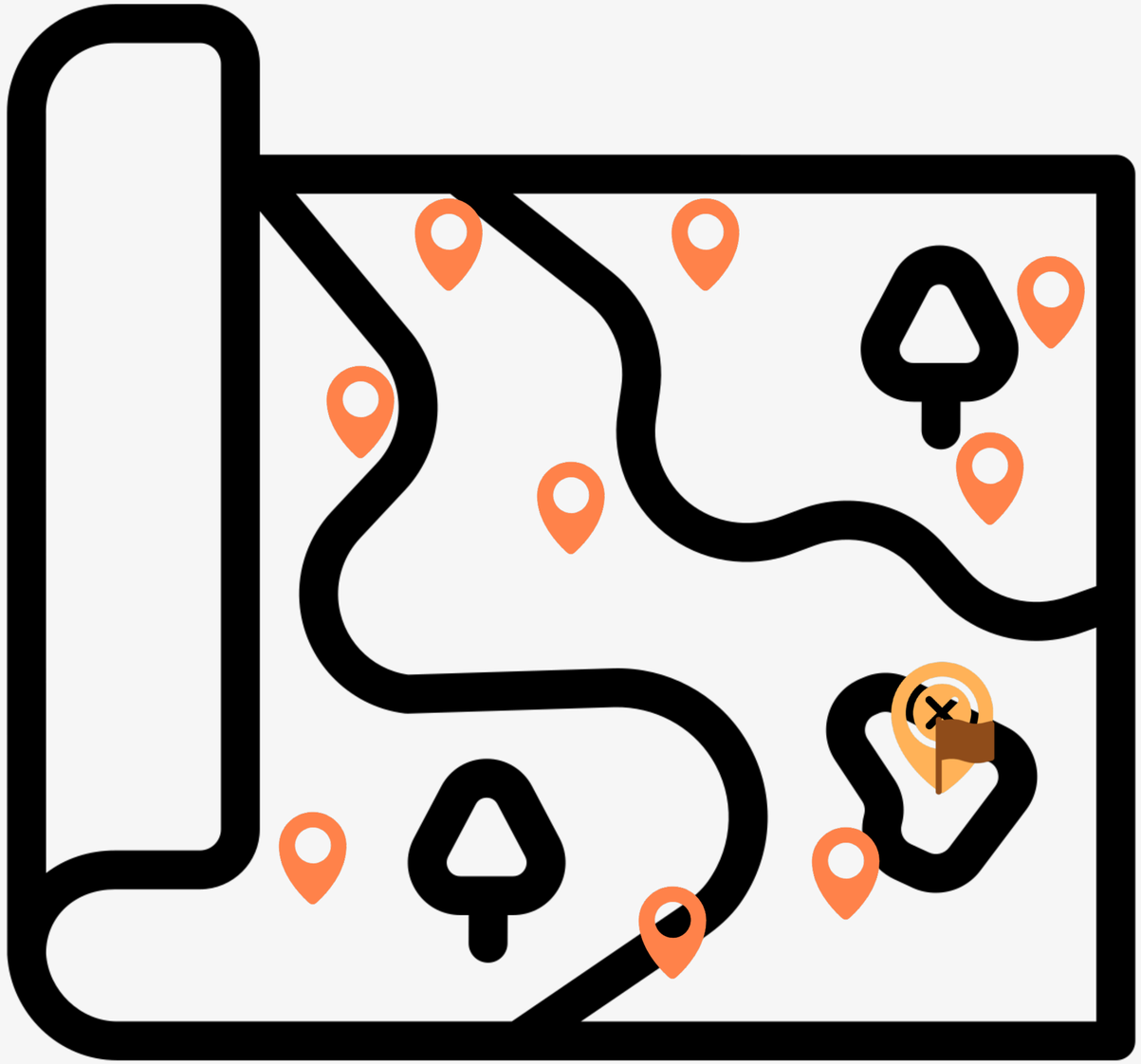

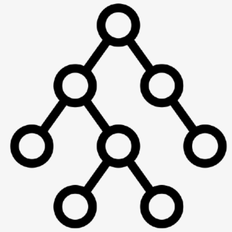

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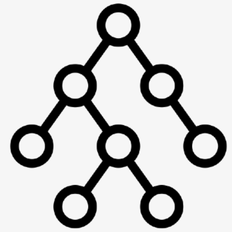

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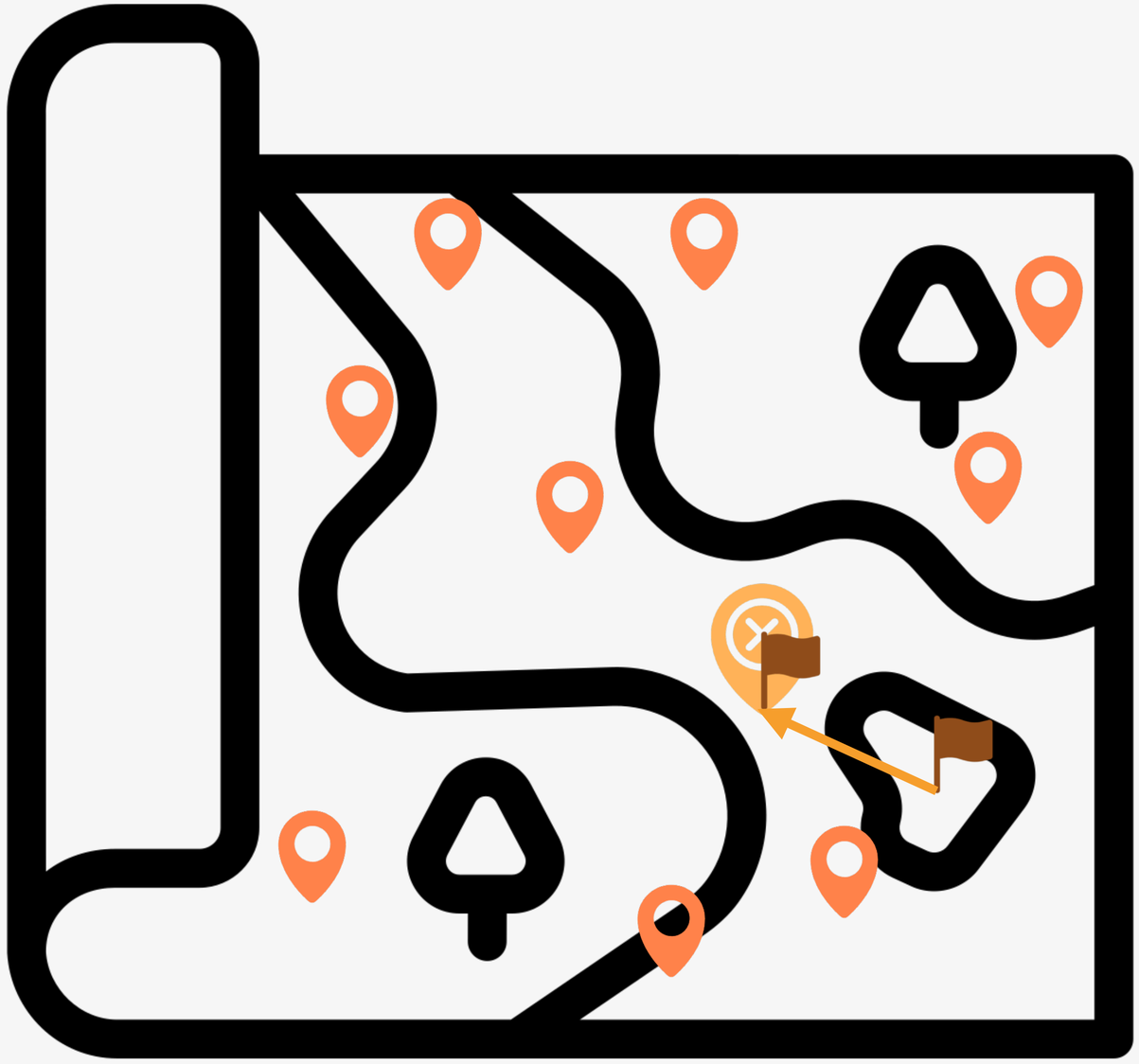

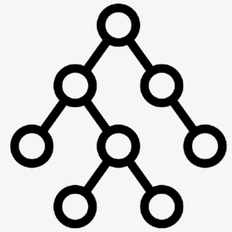

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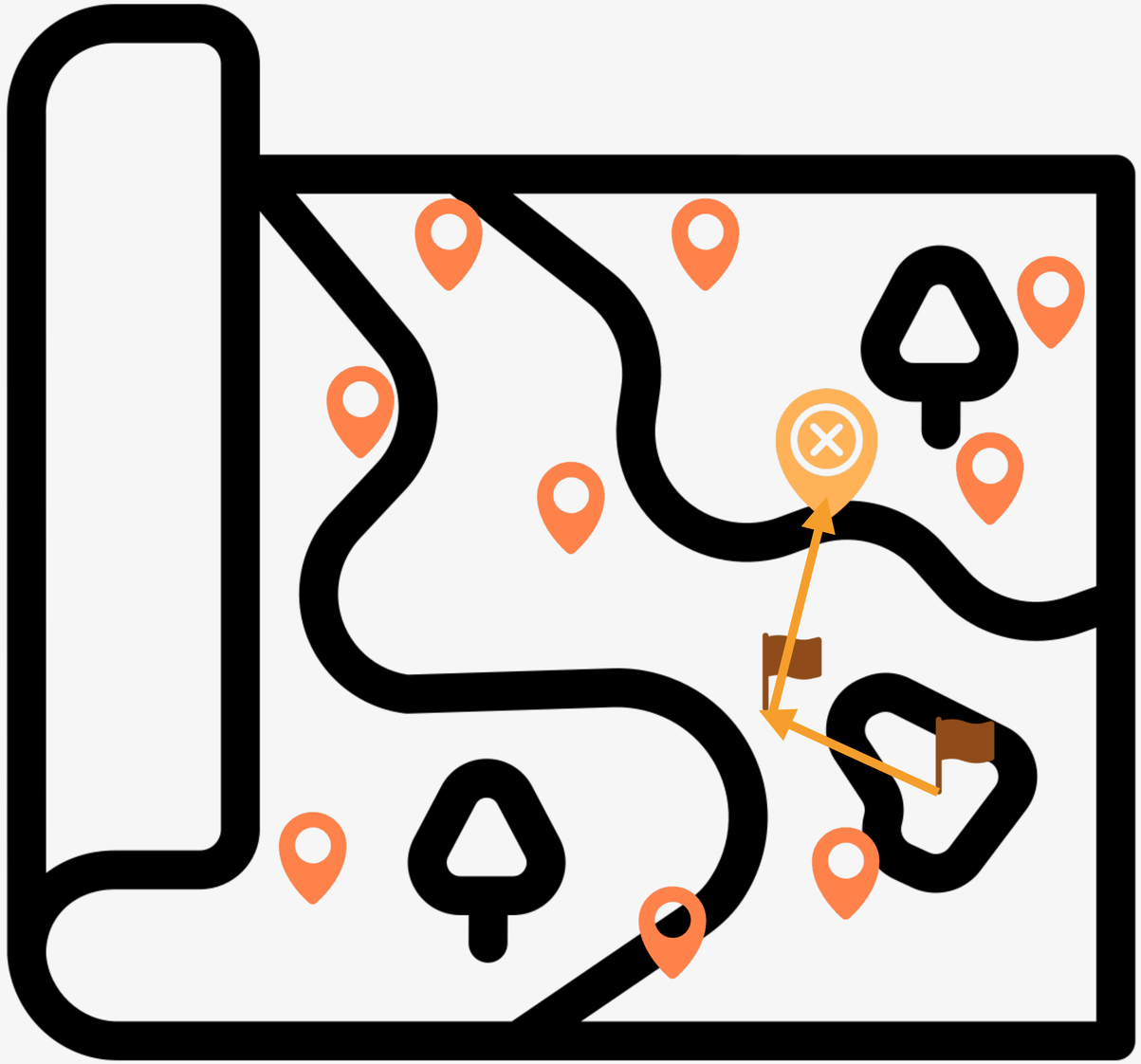

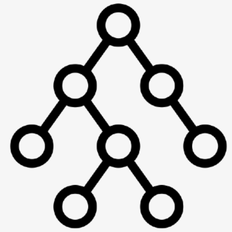

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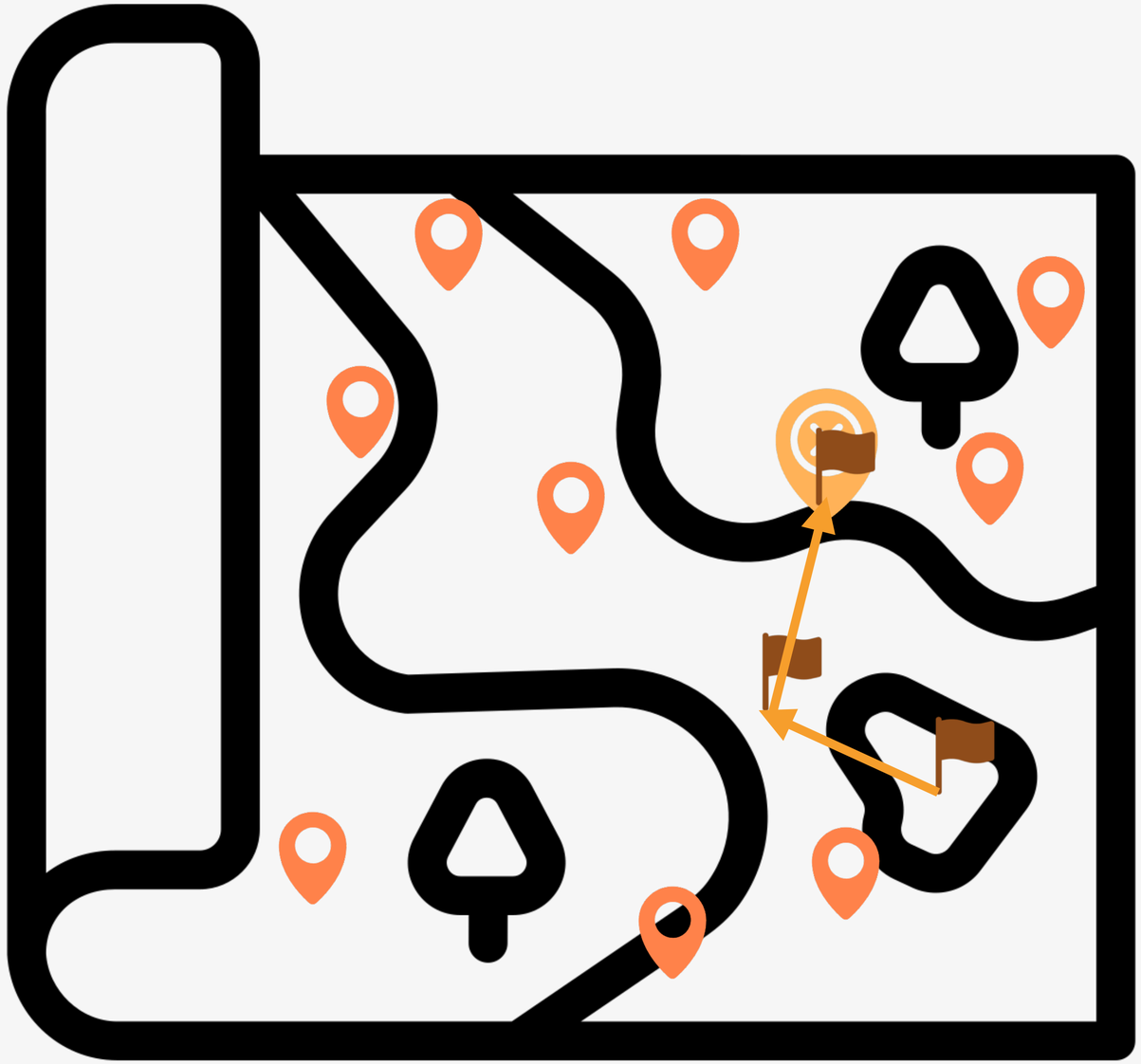

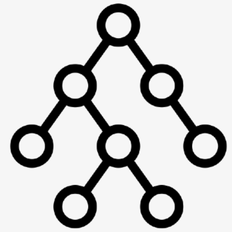

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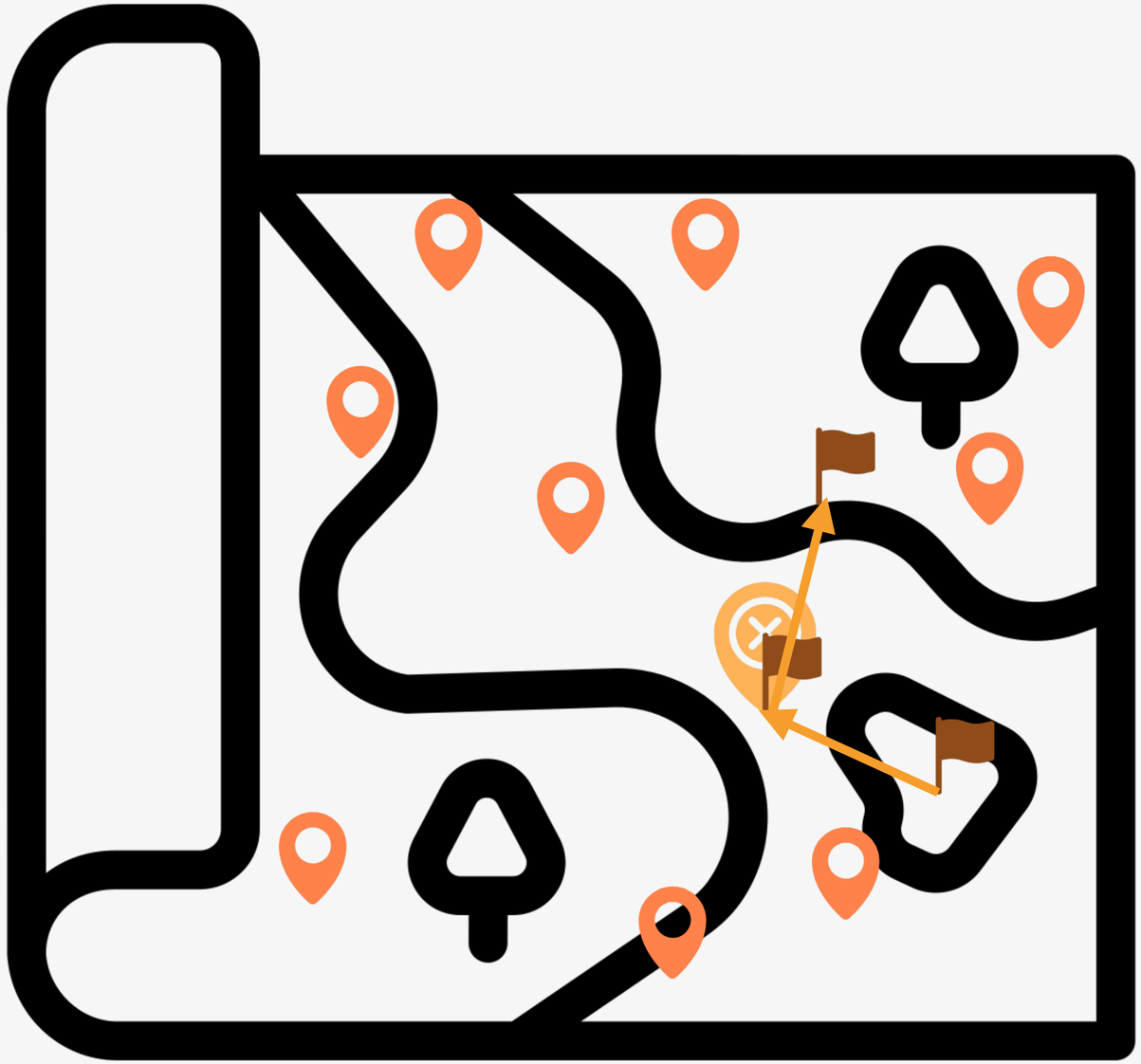

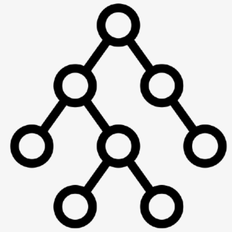

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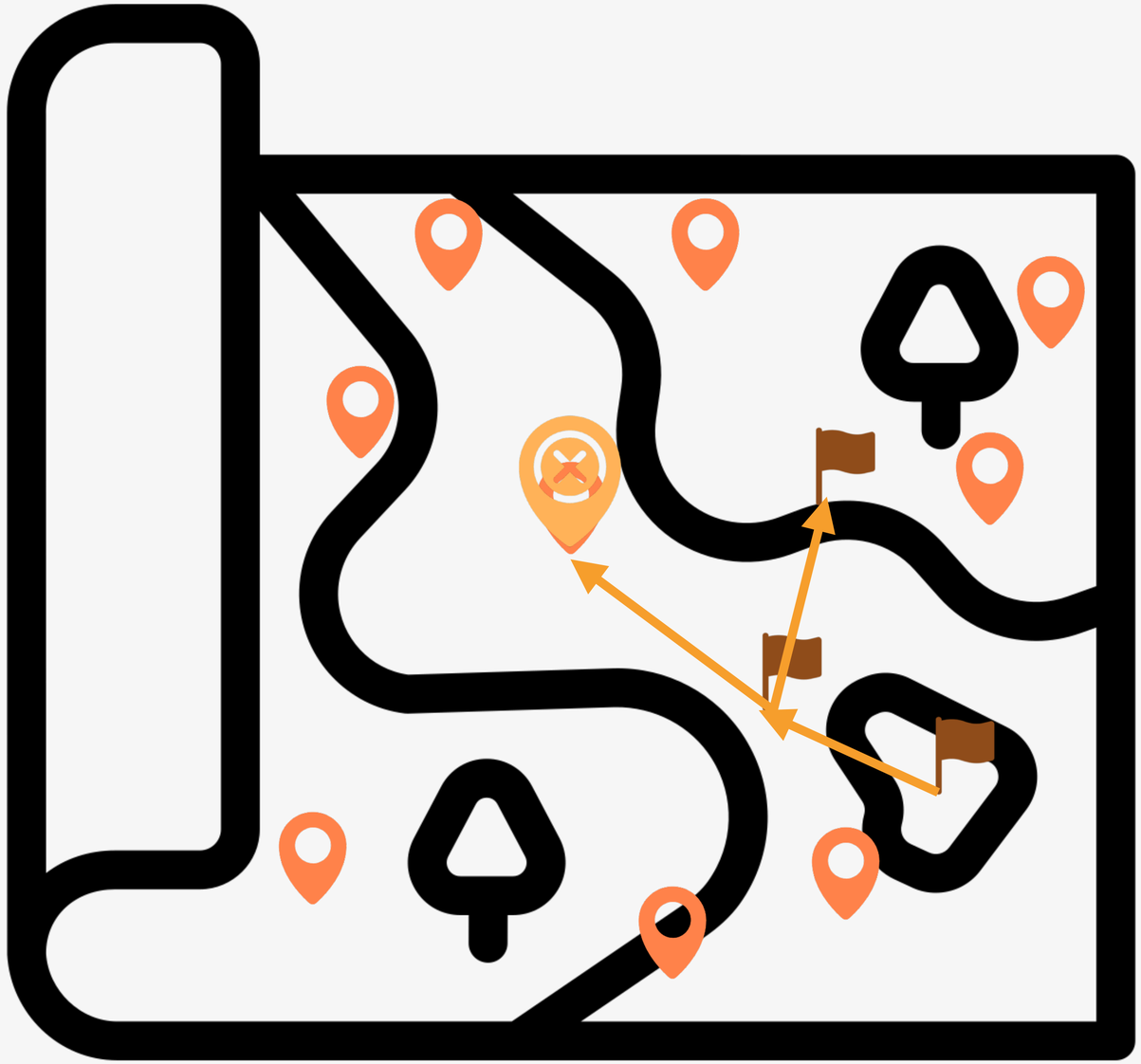

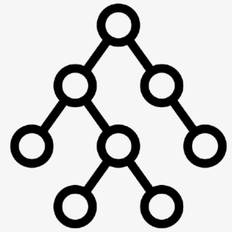

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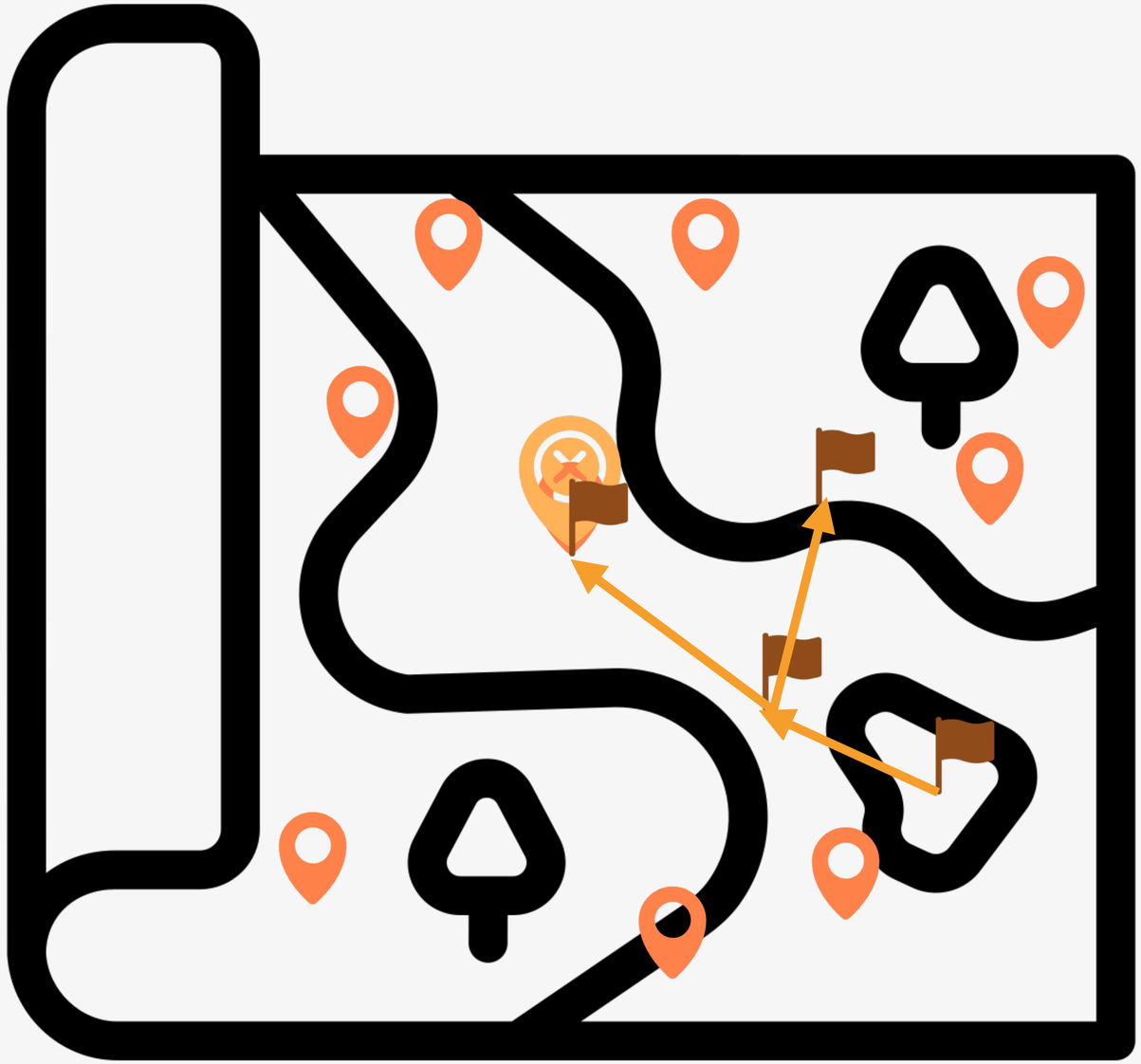

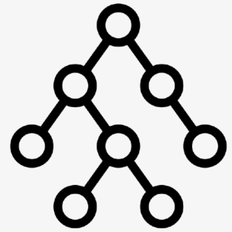

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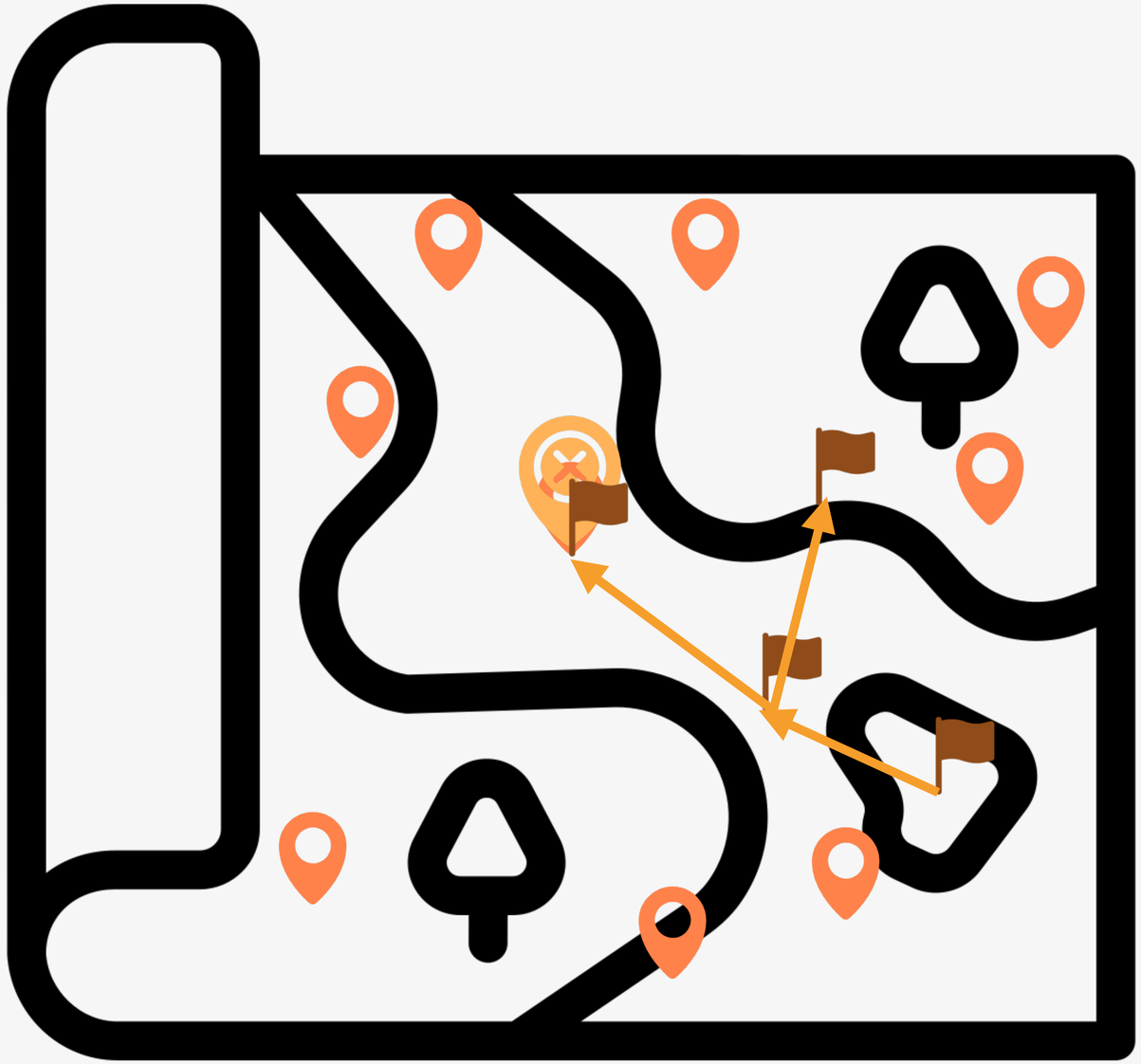

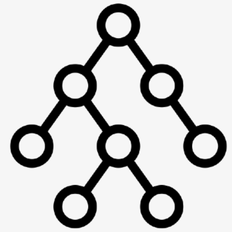

Goal of the search tree:

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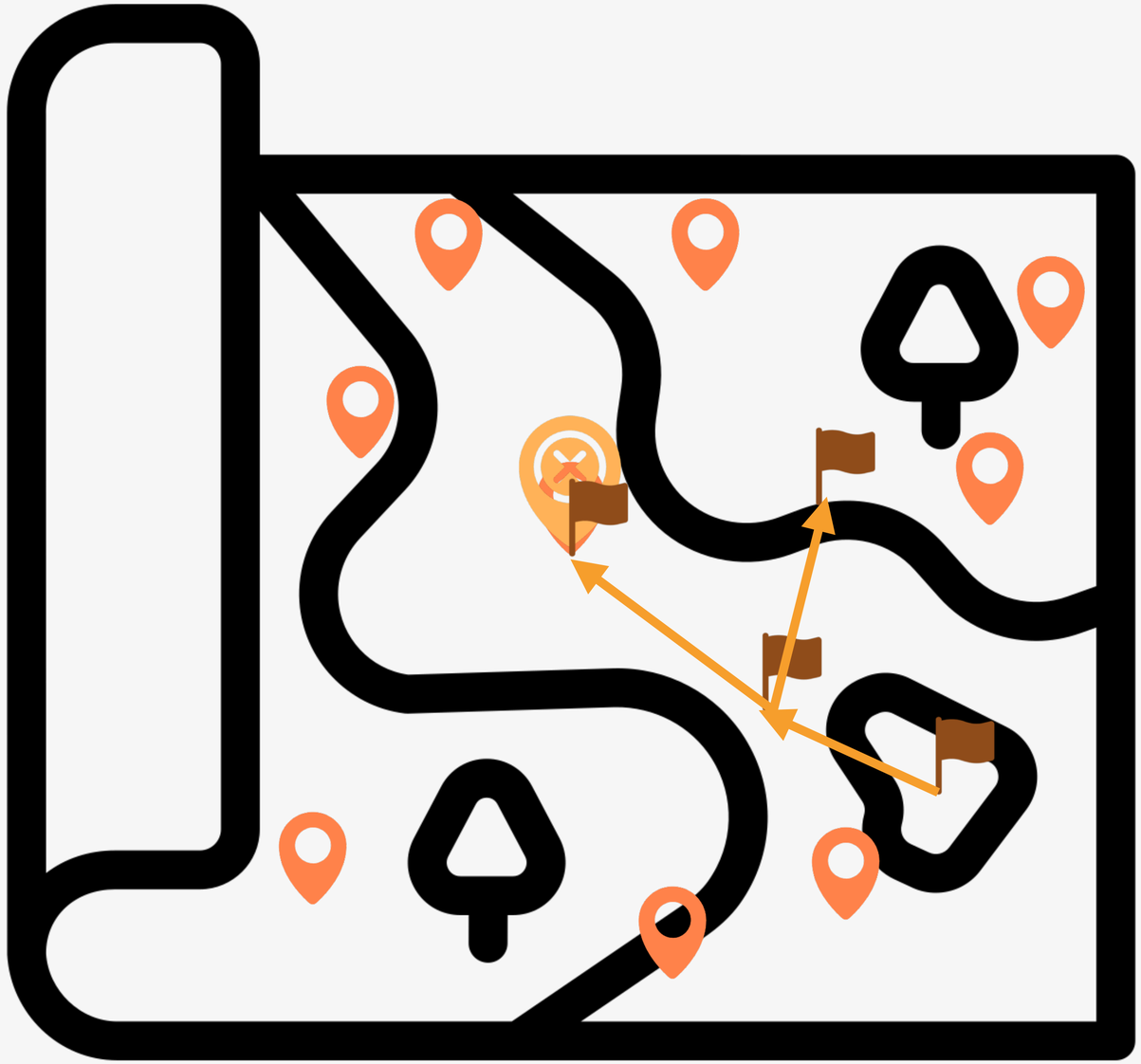
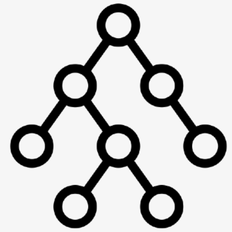
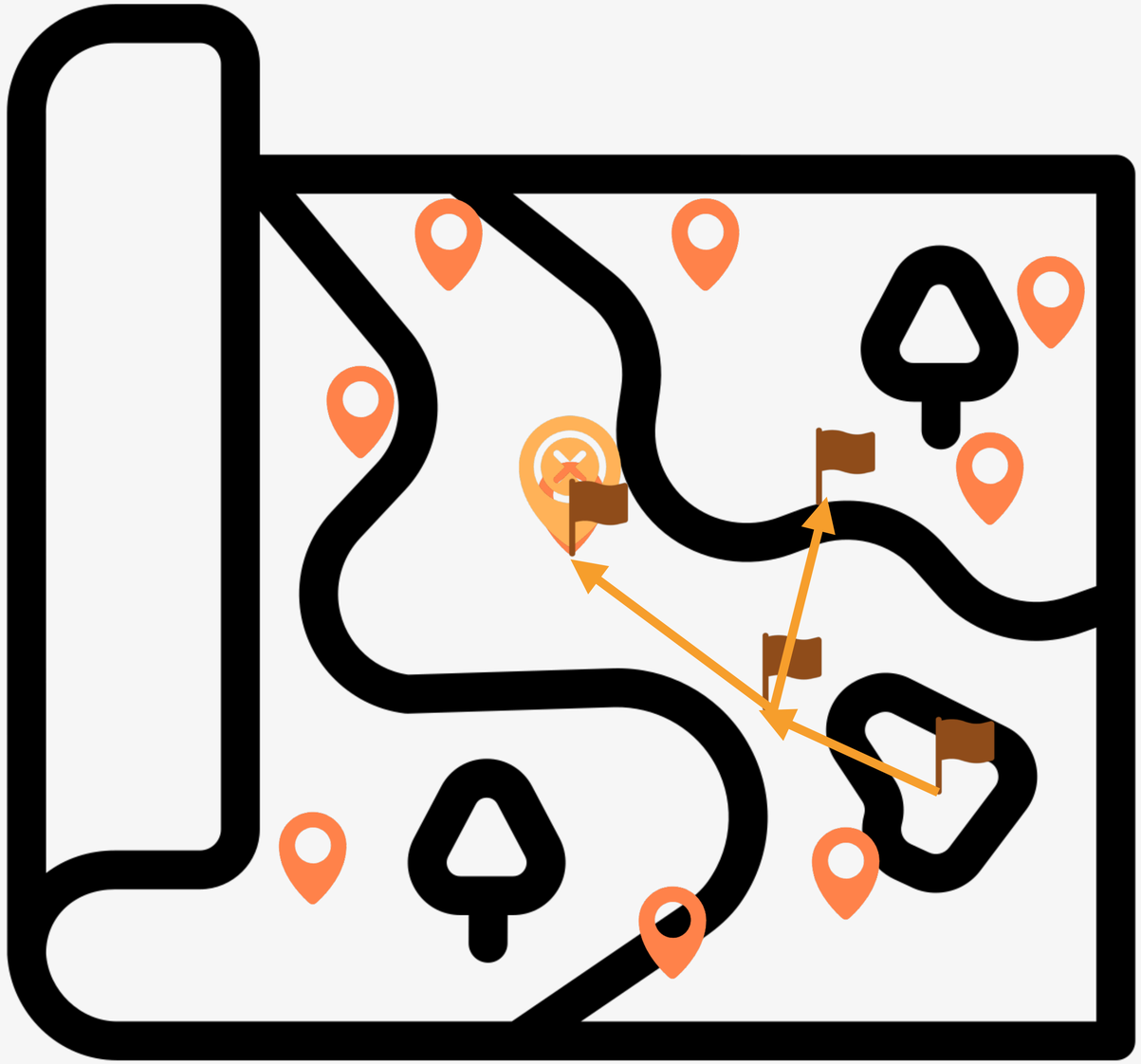


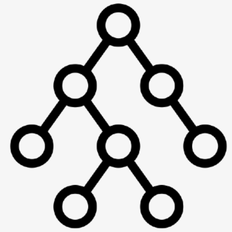


Goal of the search tree:

- **Follow the search at each step**

SEARCH TREE

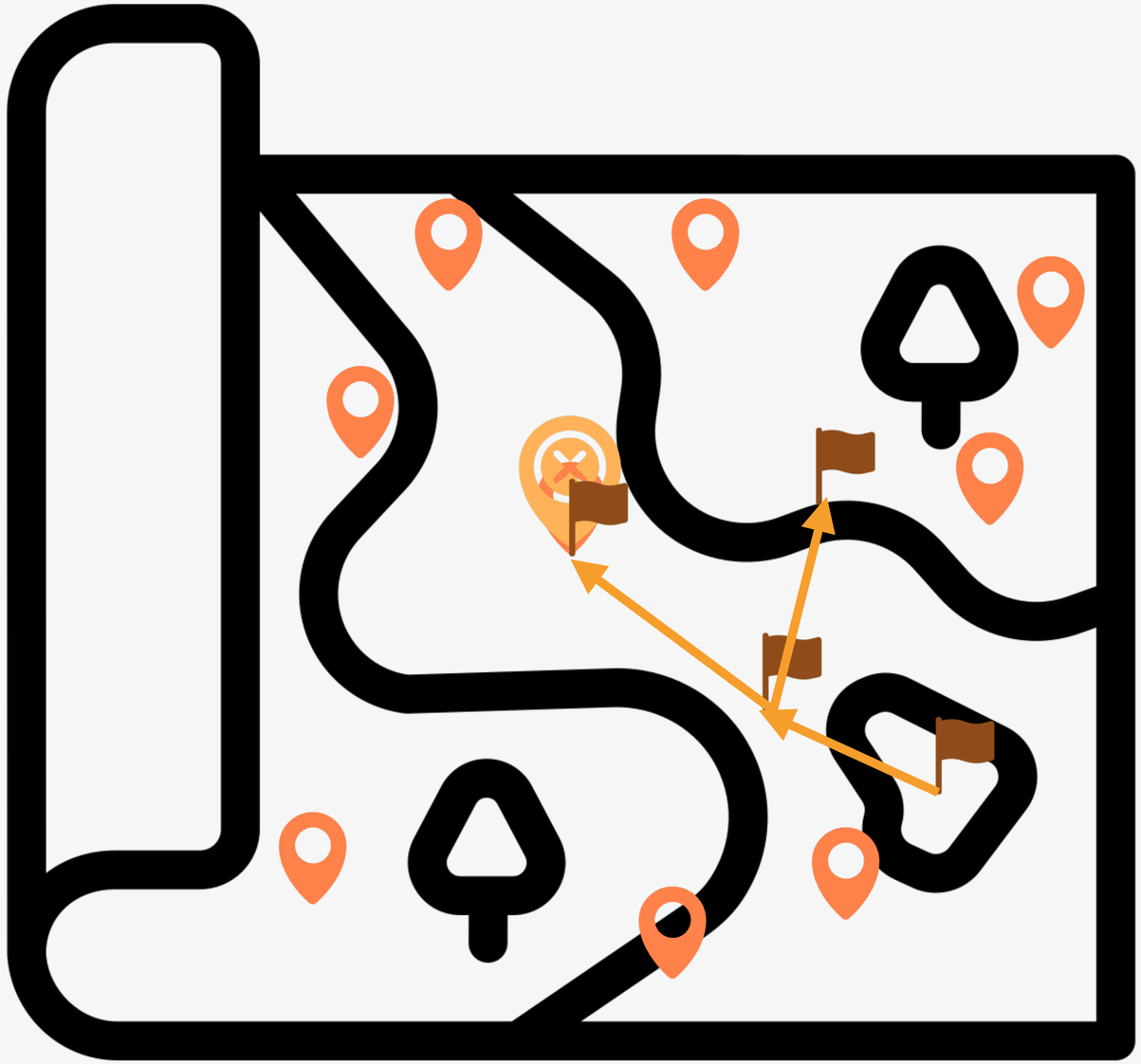


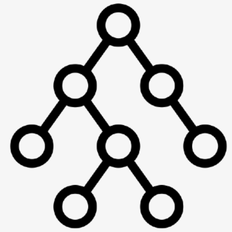


Goal of the search tree:

- **Follow the search at each step**
- **Call the fix-point after each step**

SEARCH TREE





Goal of the search tree:

- Follow the search at each step
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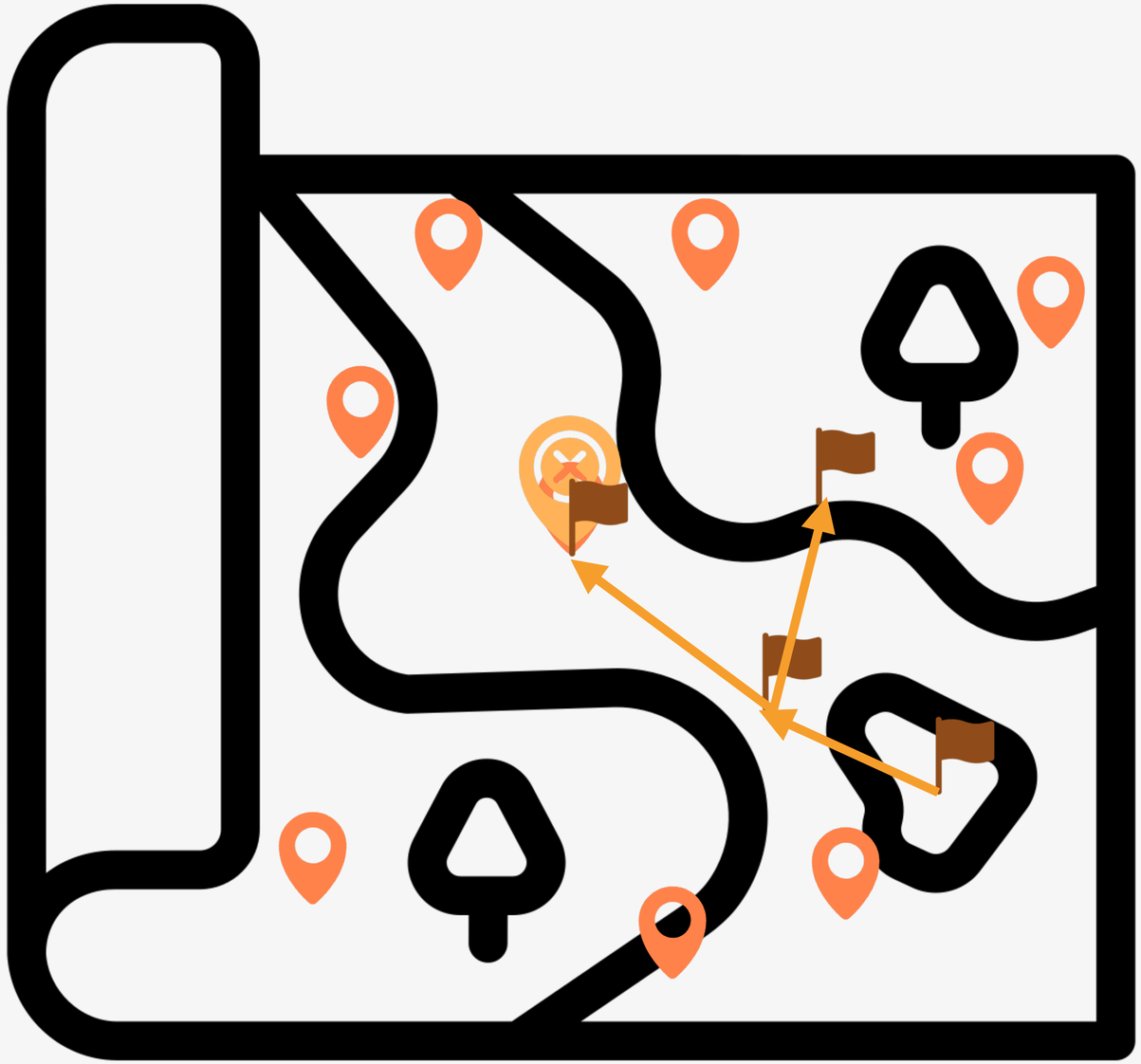
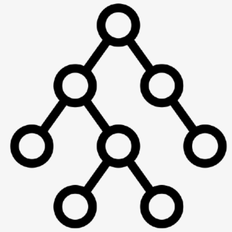
- Put the save point in order to go back

SEARCH TREE





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- Not explore twice the same places

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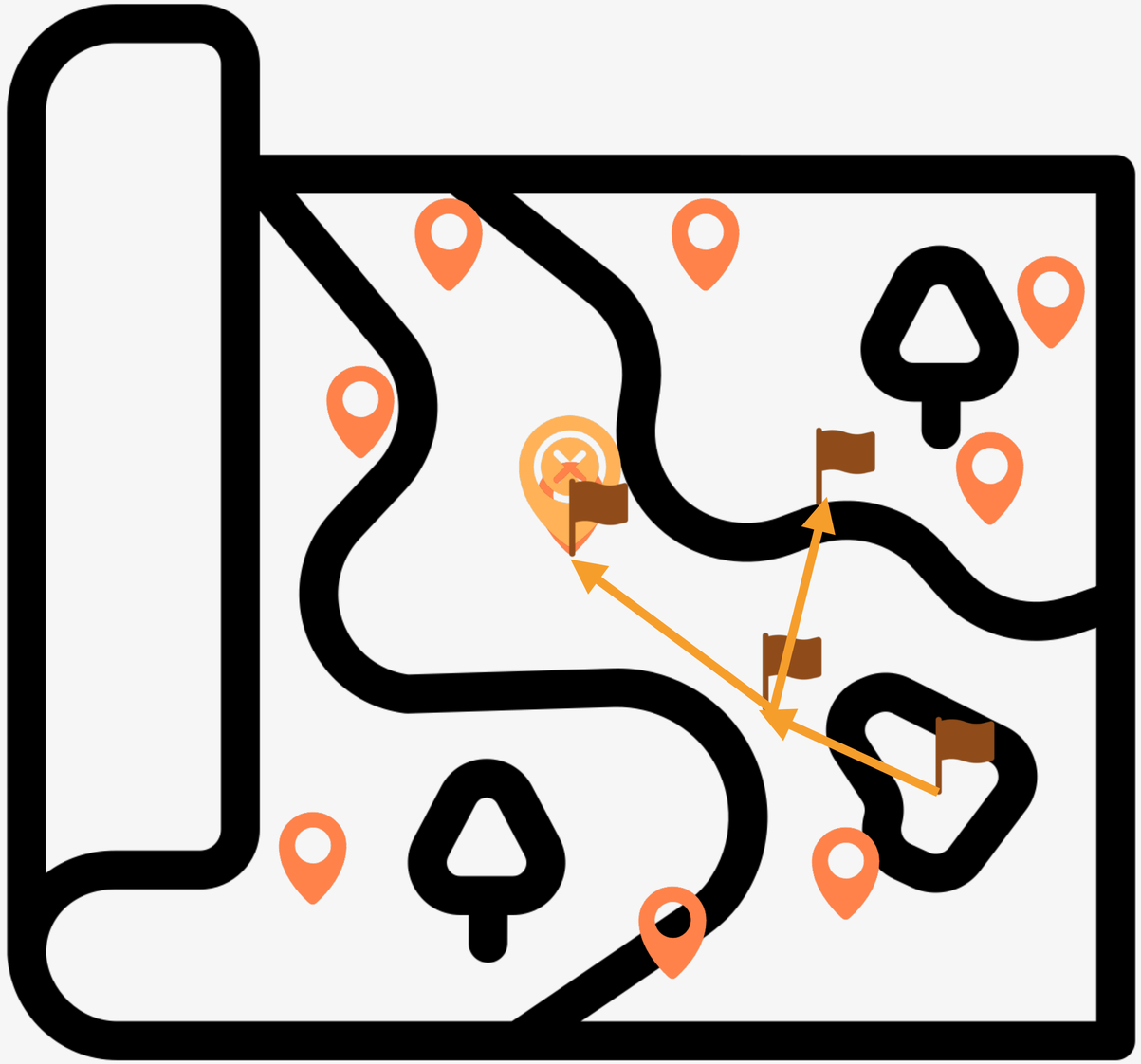

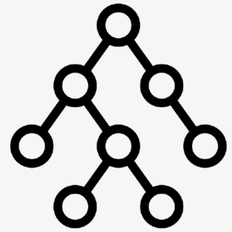

Goal of the search tree:

- Follow the search at each step
- Call the fix-point after each step
- Put the save point in order to go back
- Not explore twice the same places
- Explore until goal is reached (i.e., a solution, all solutions, optimal solution)

Exemple: Solving of a Minimisation ILP With Branch and Bound

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UB: $+\infty$

Exemple: Solving of a Minimisation ILP With Branch and Bound

UB:+∞

LB:3.5

Exemple: Solving of a Minimisation ILP With Branch and Bound

UB:+∞

LB:3.5

(X=3.4,Y=2.3)

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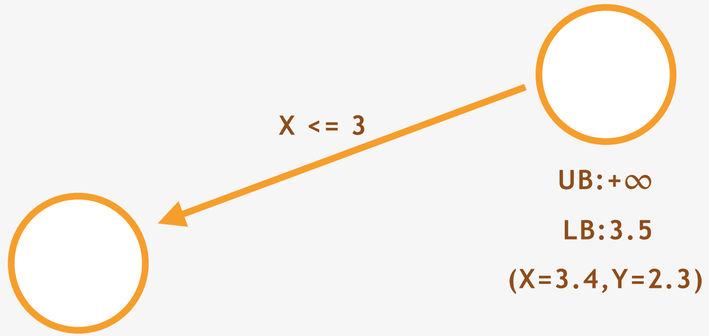

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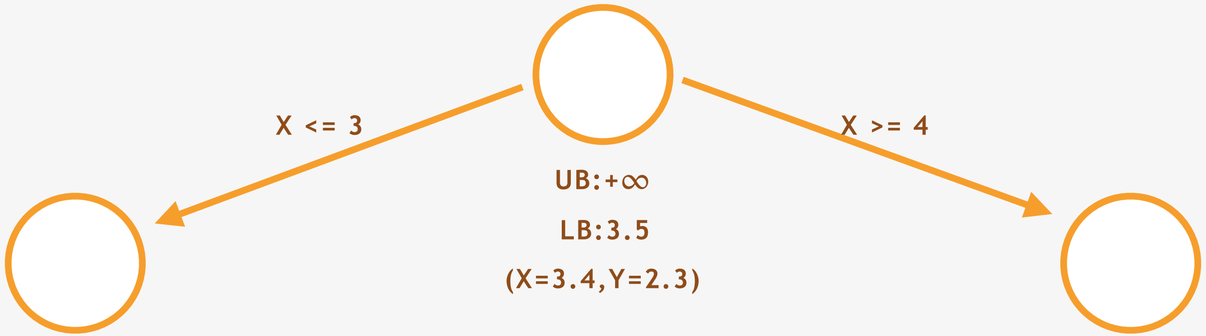

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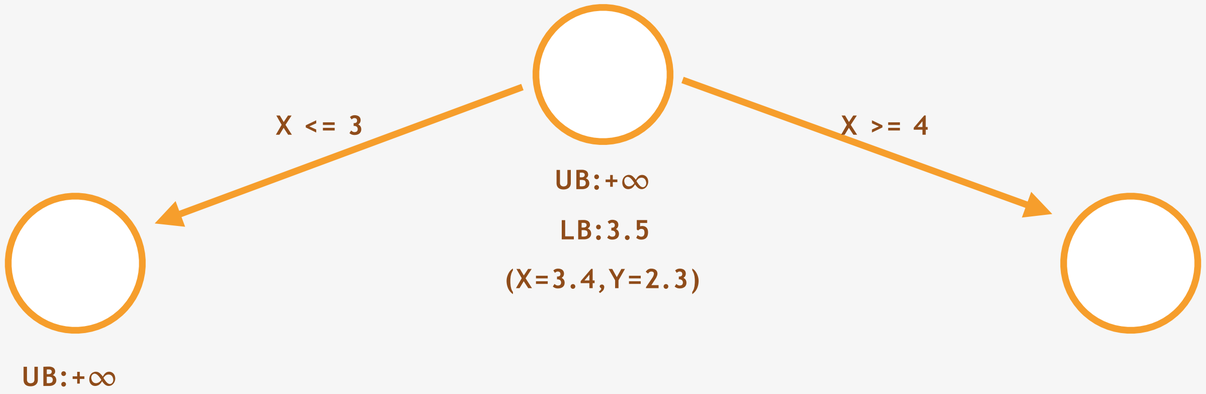
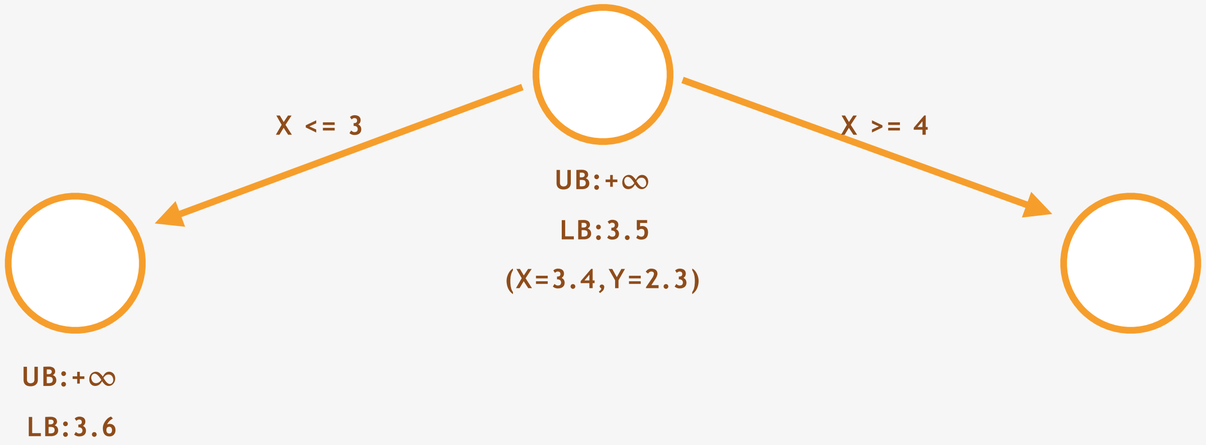


COUSIN OF THE BRANCH-AND-BOUND

Exemple: Solving of a Minimisation ILP With Branch and Bound



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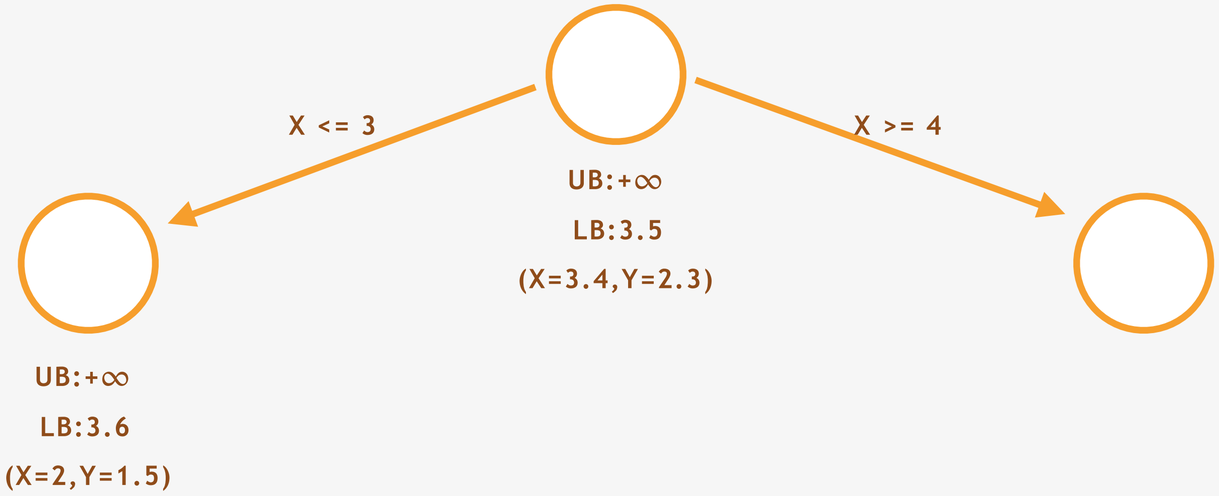

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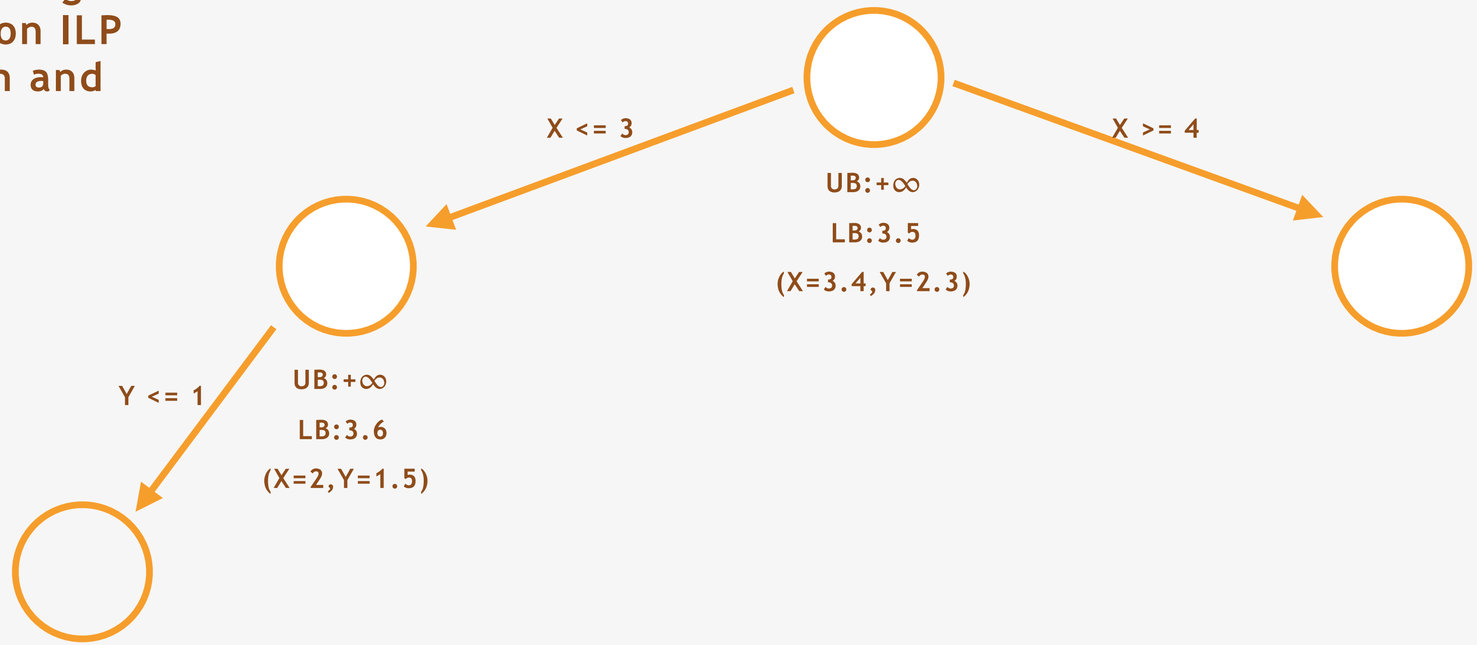

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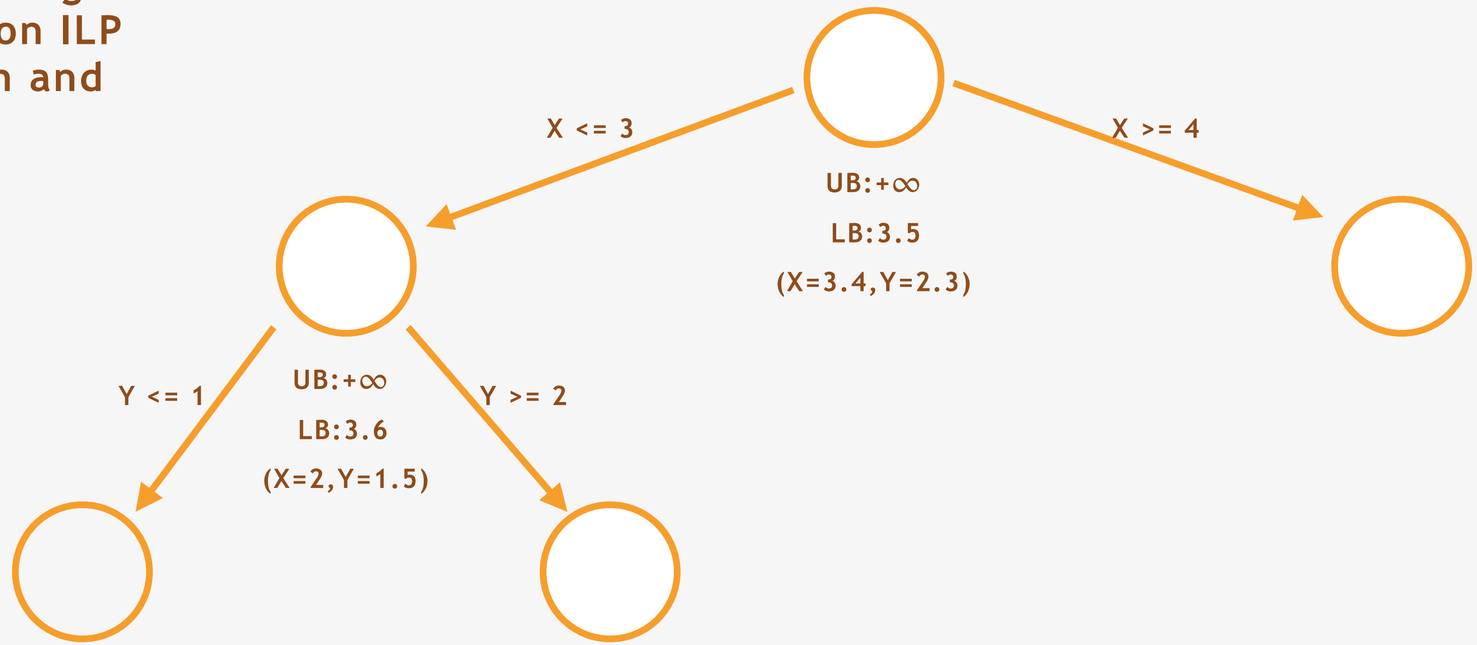

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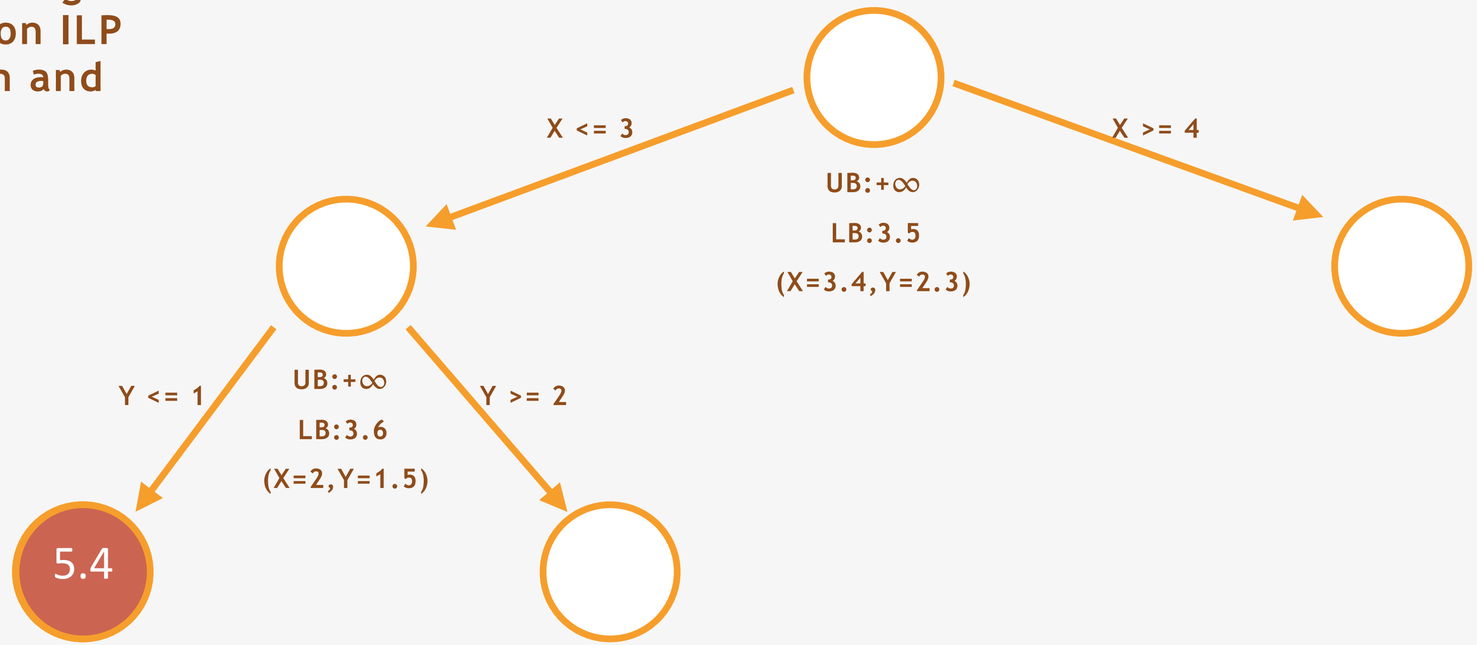

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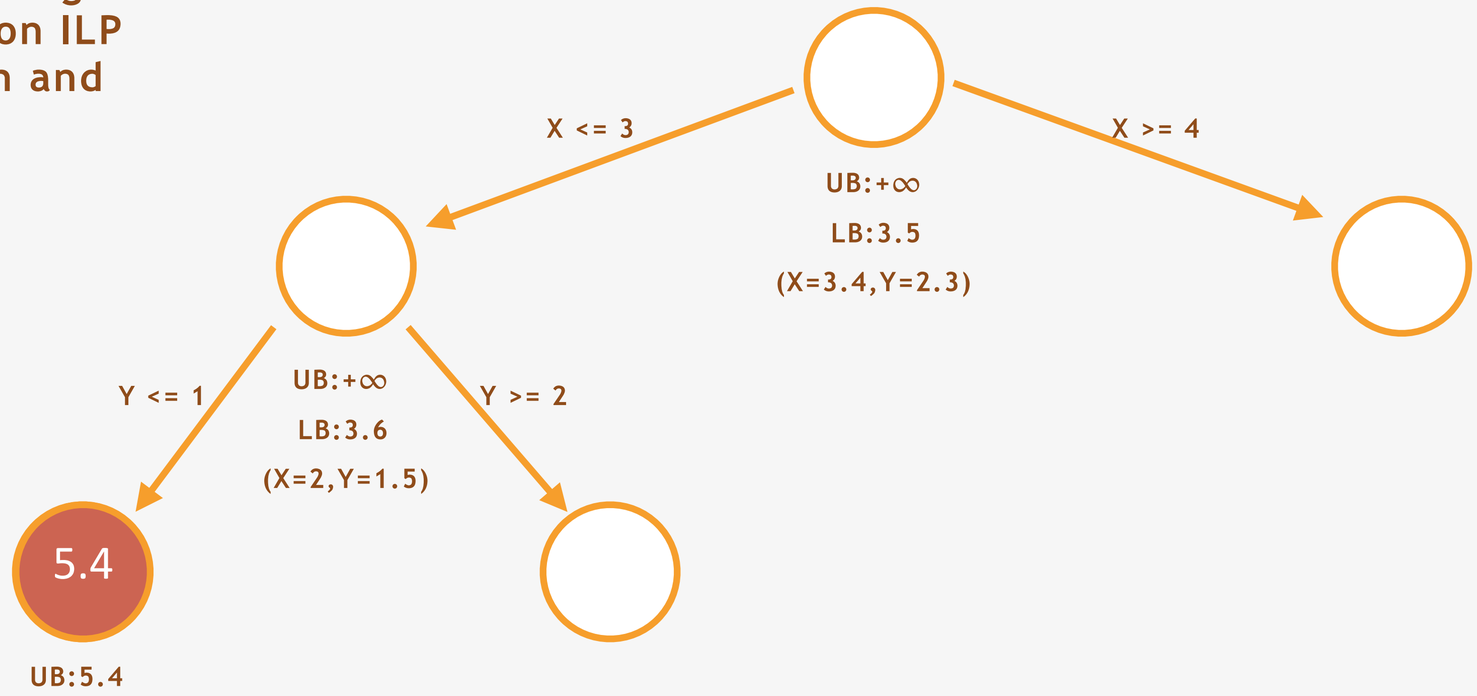

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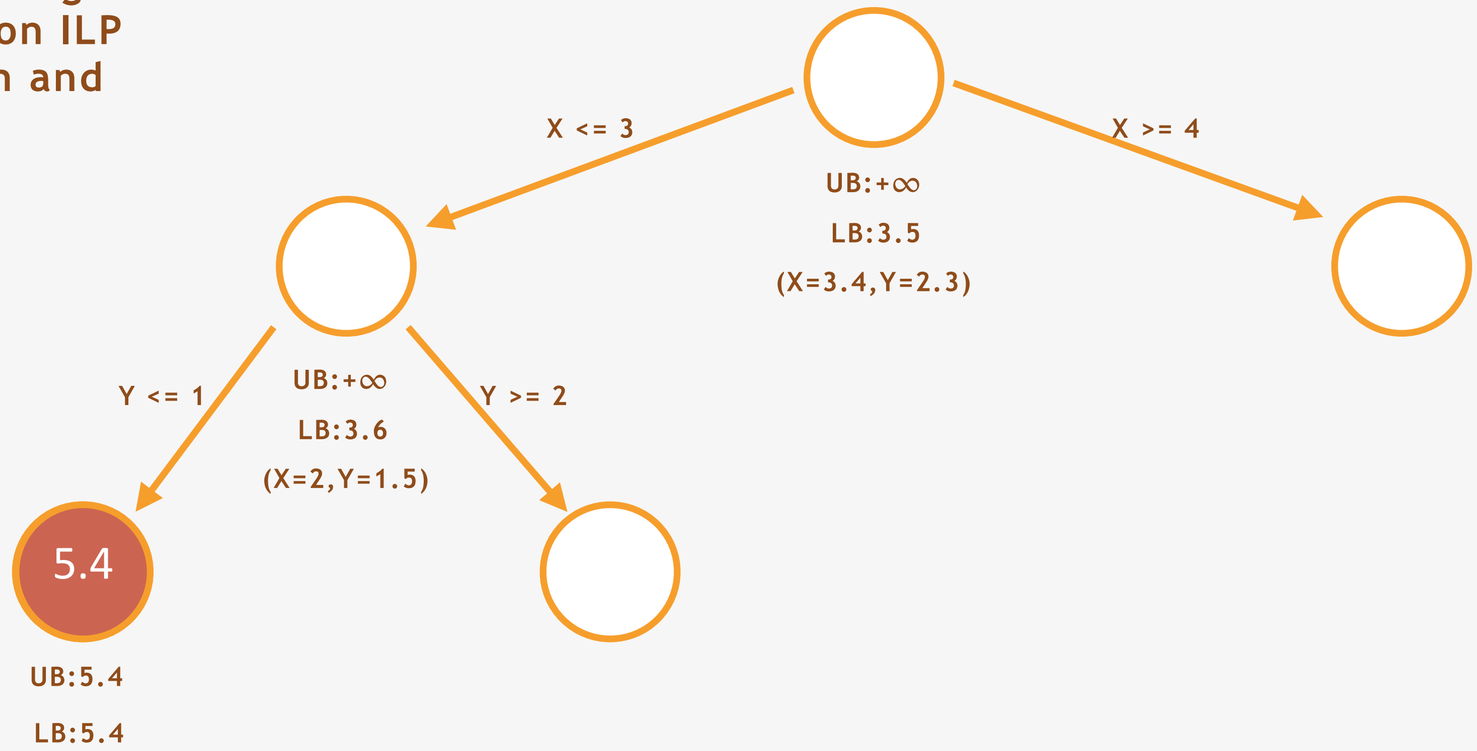

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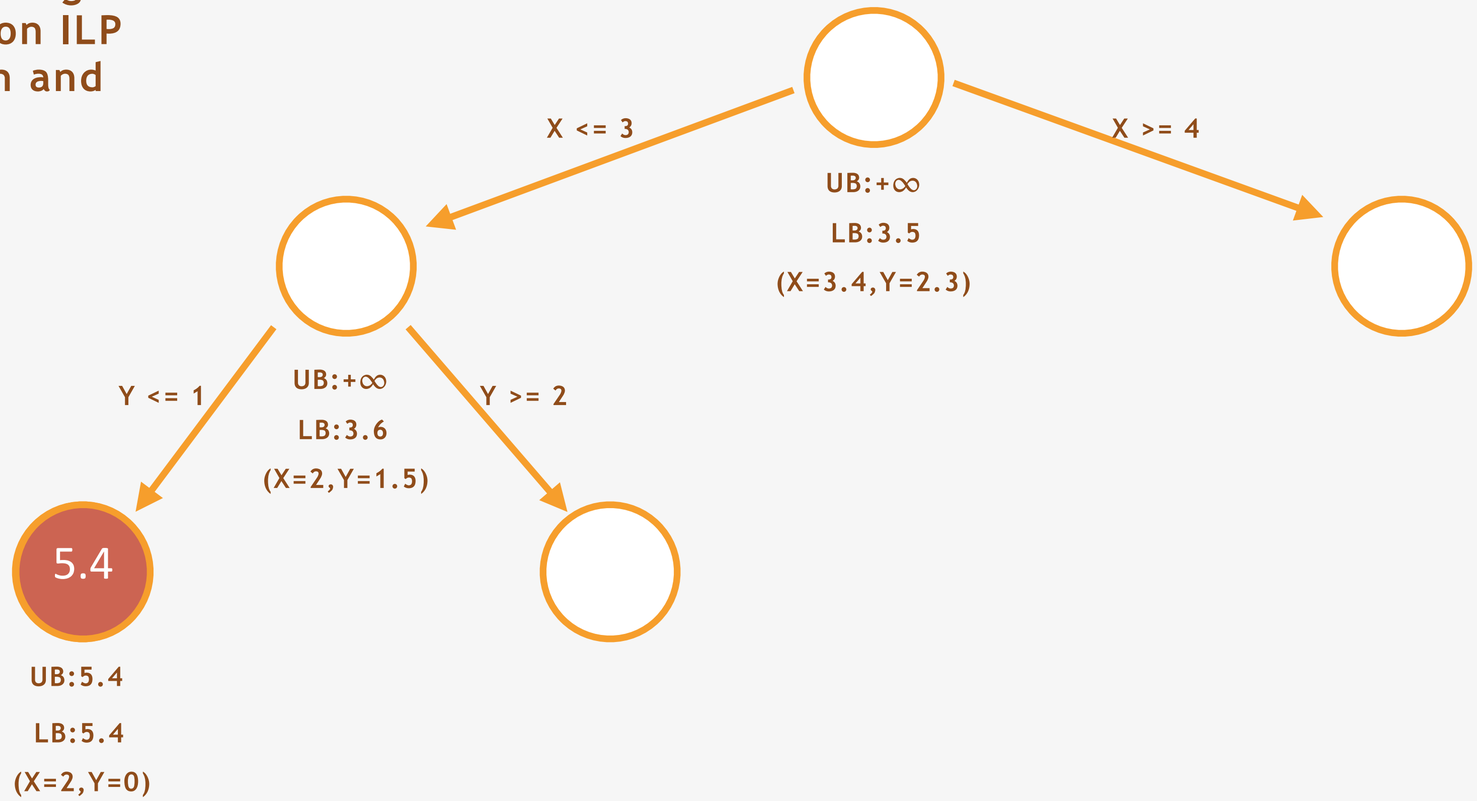

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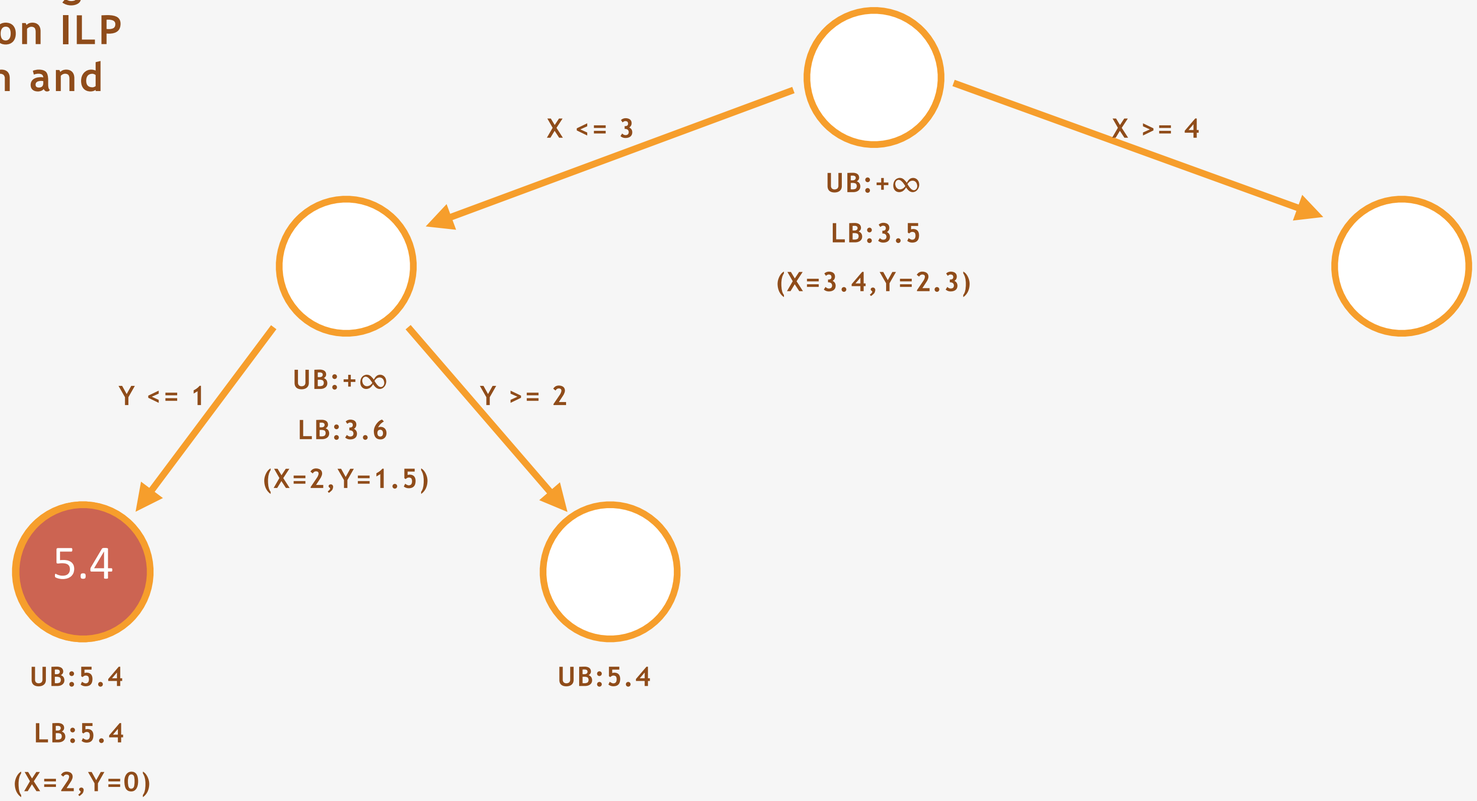

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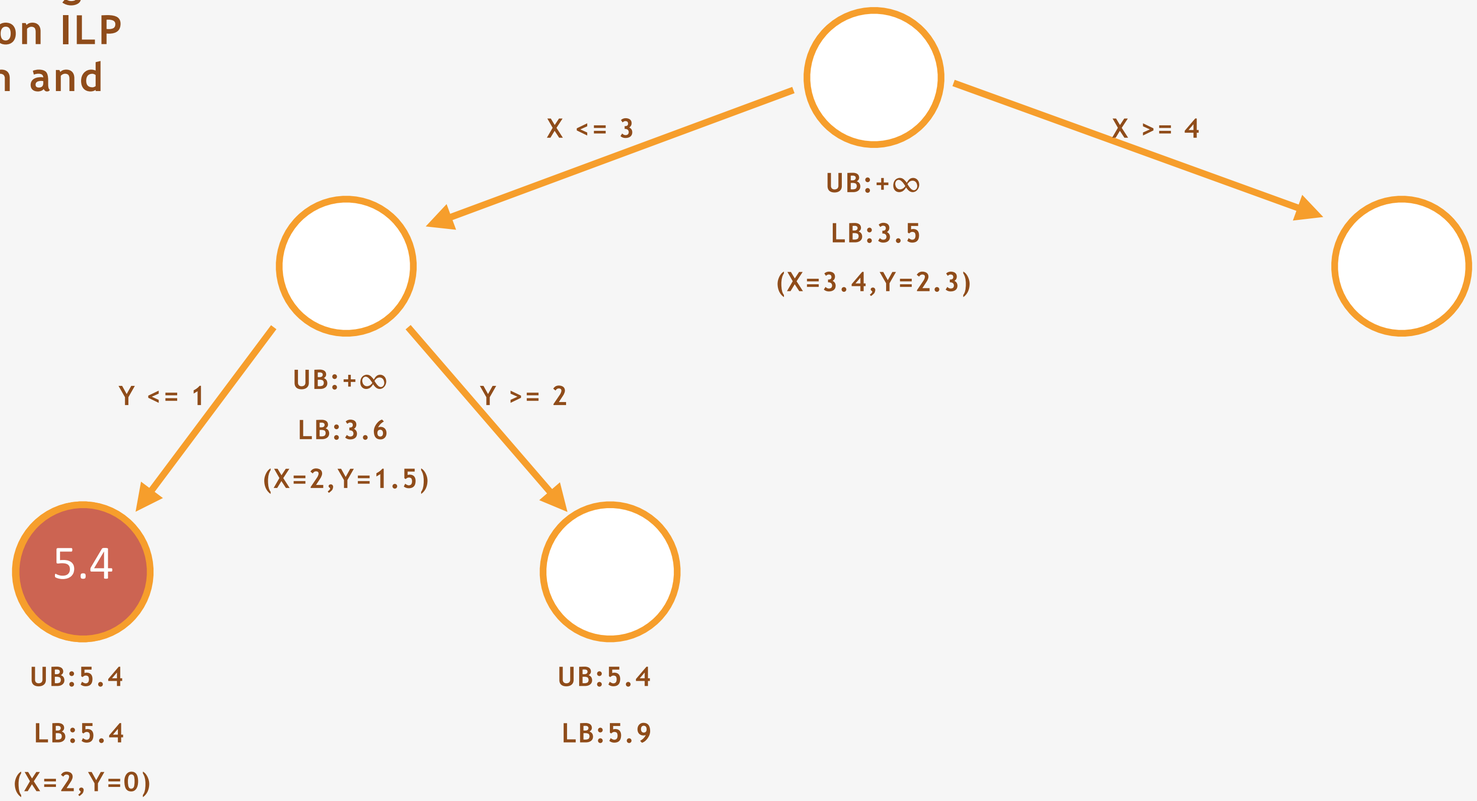

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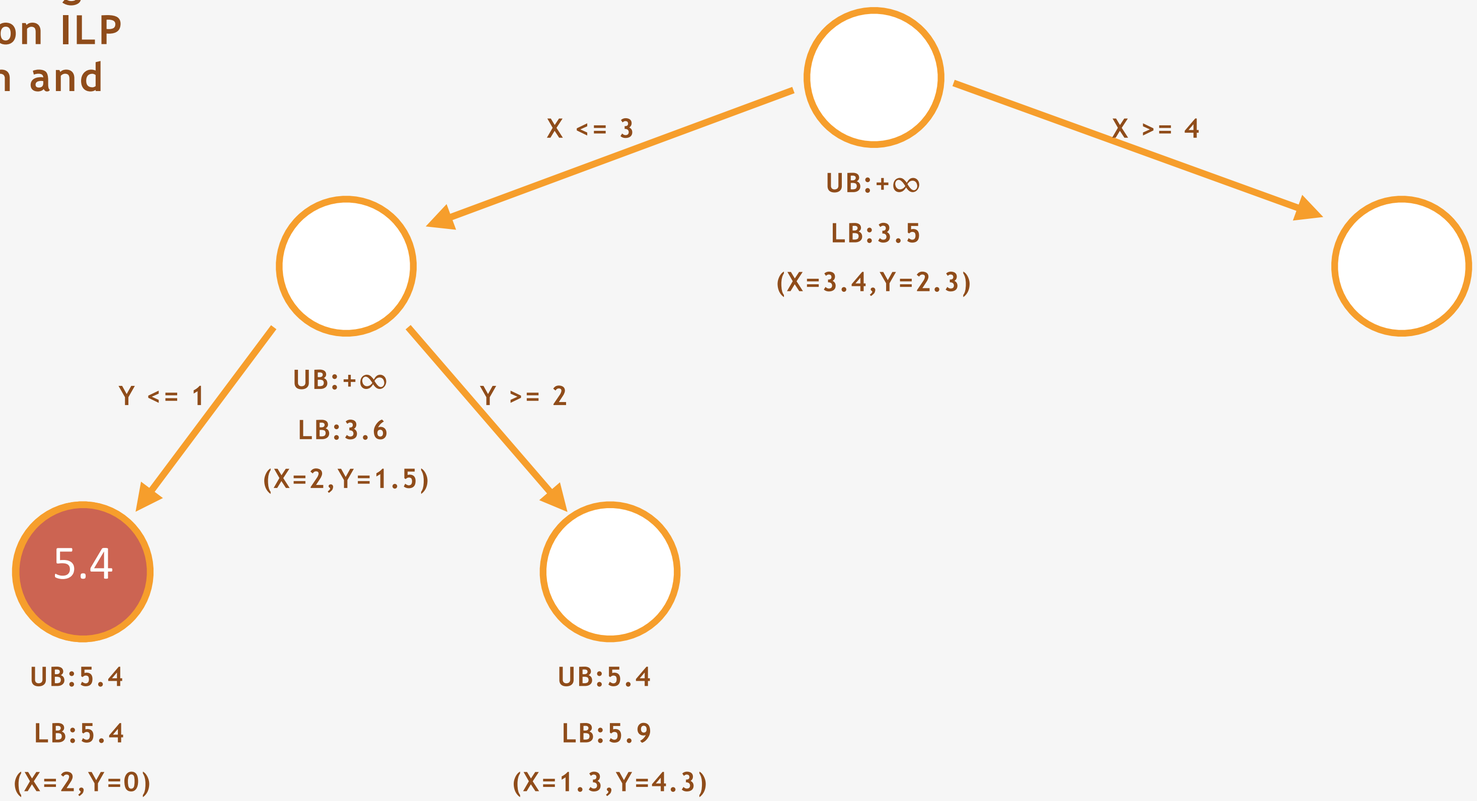
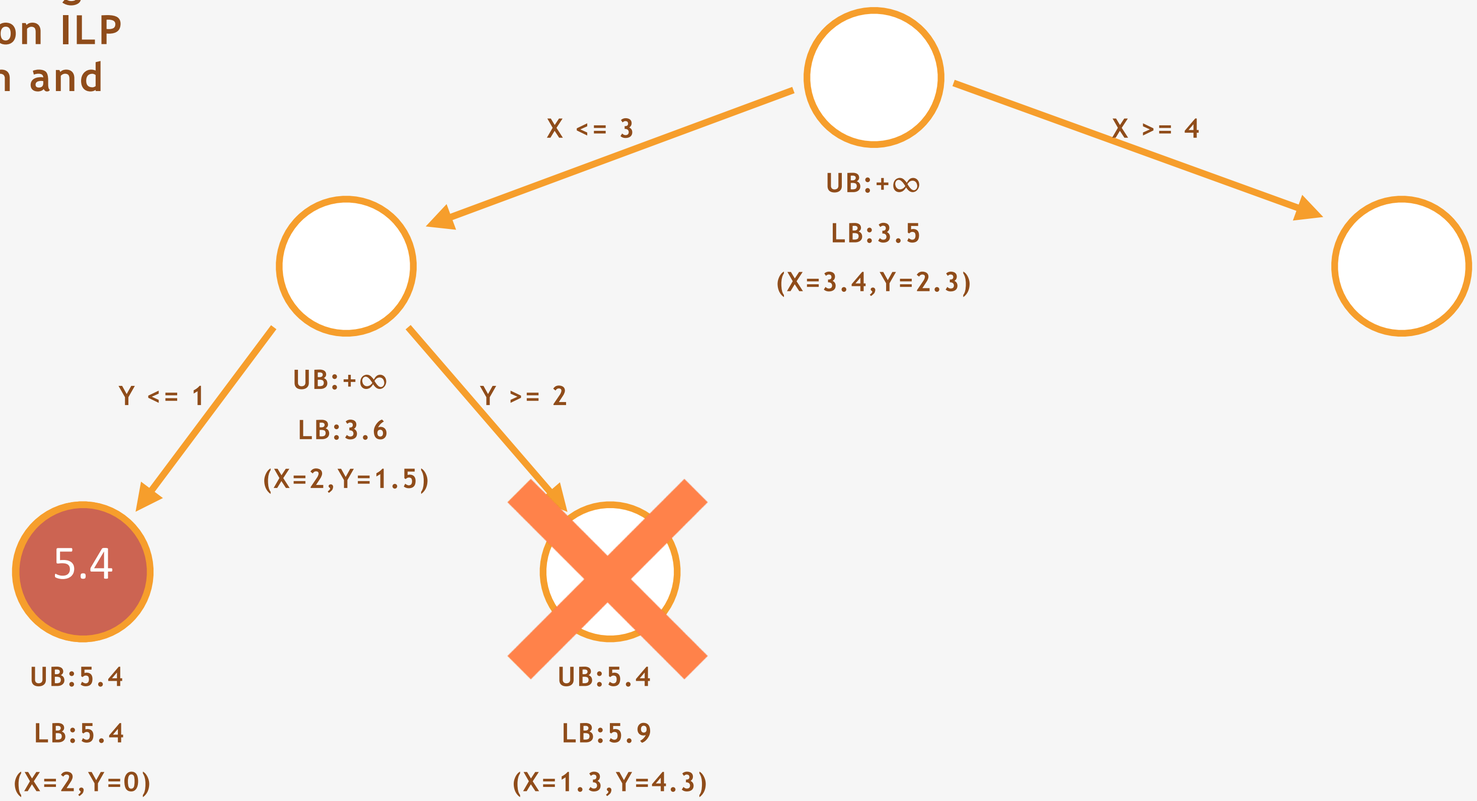


Exemple: Solving of a Minimisation ILP With Branch and Bound



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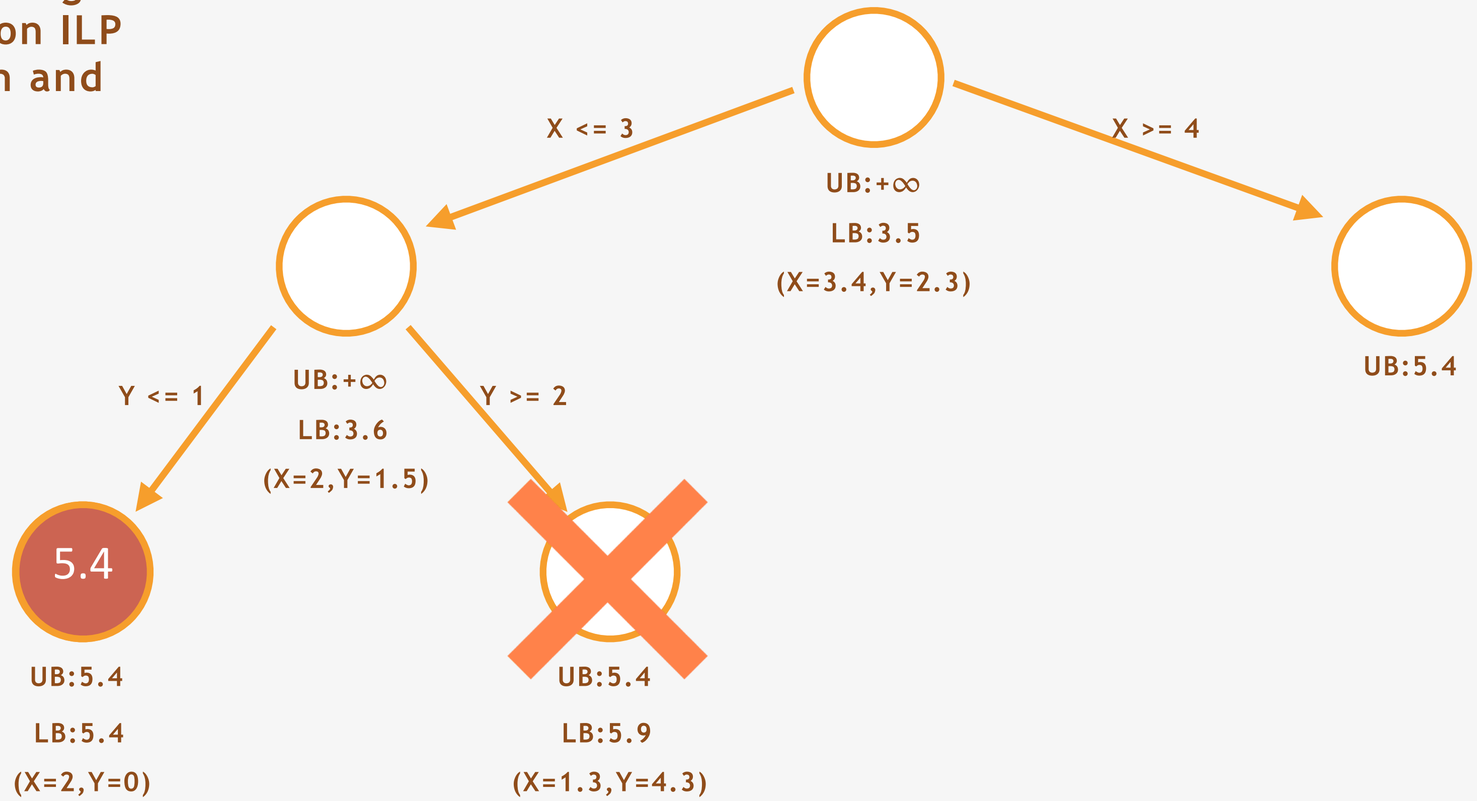

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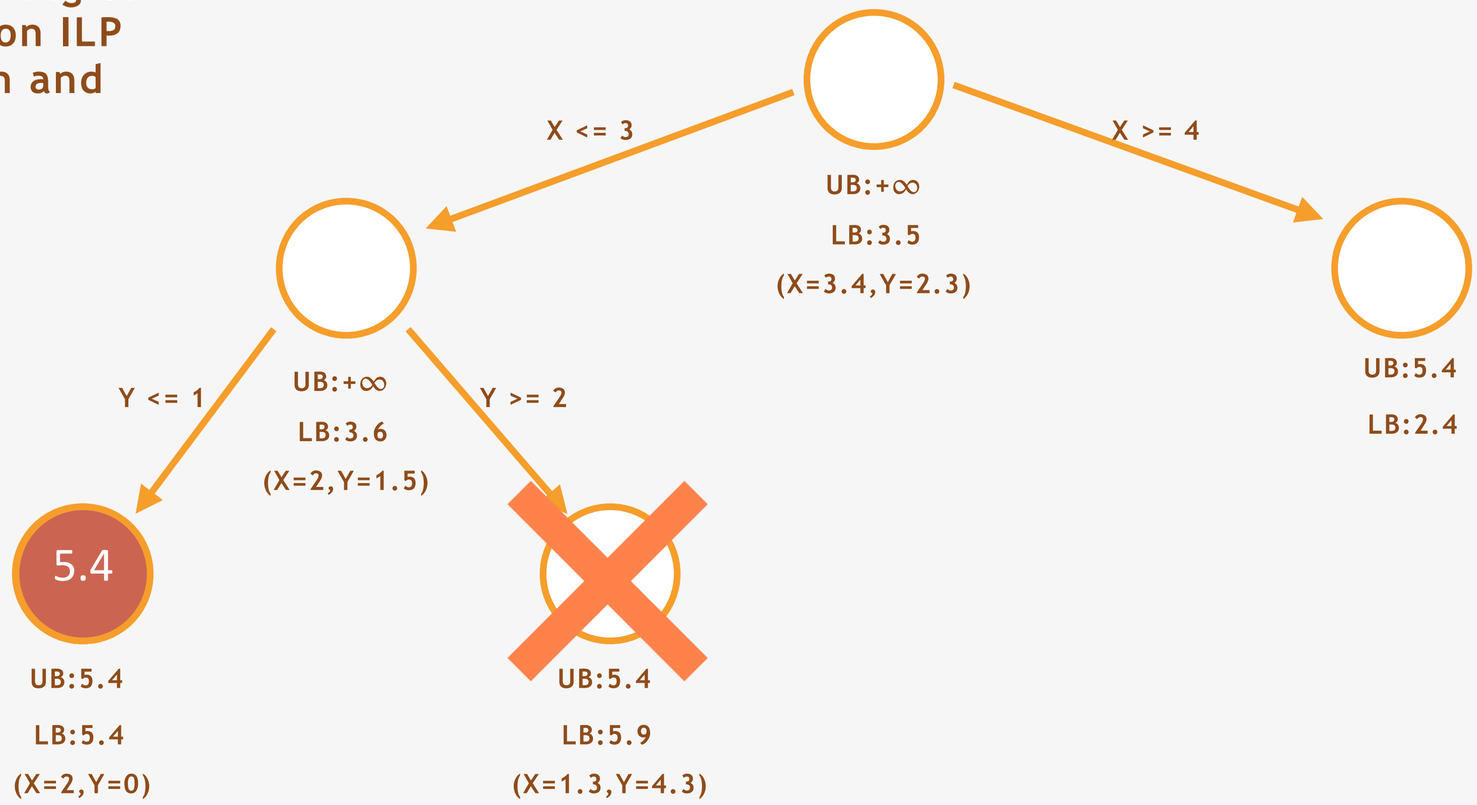

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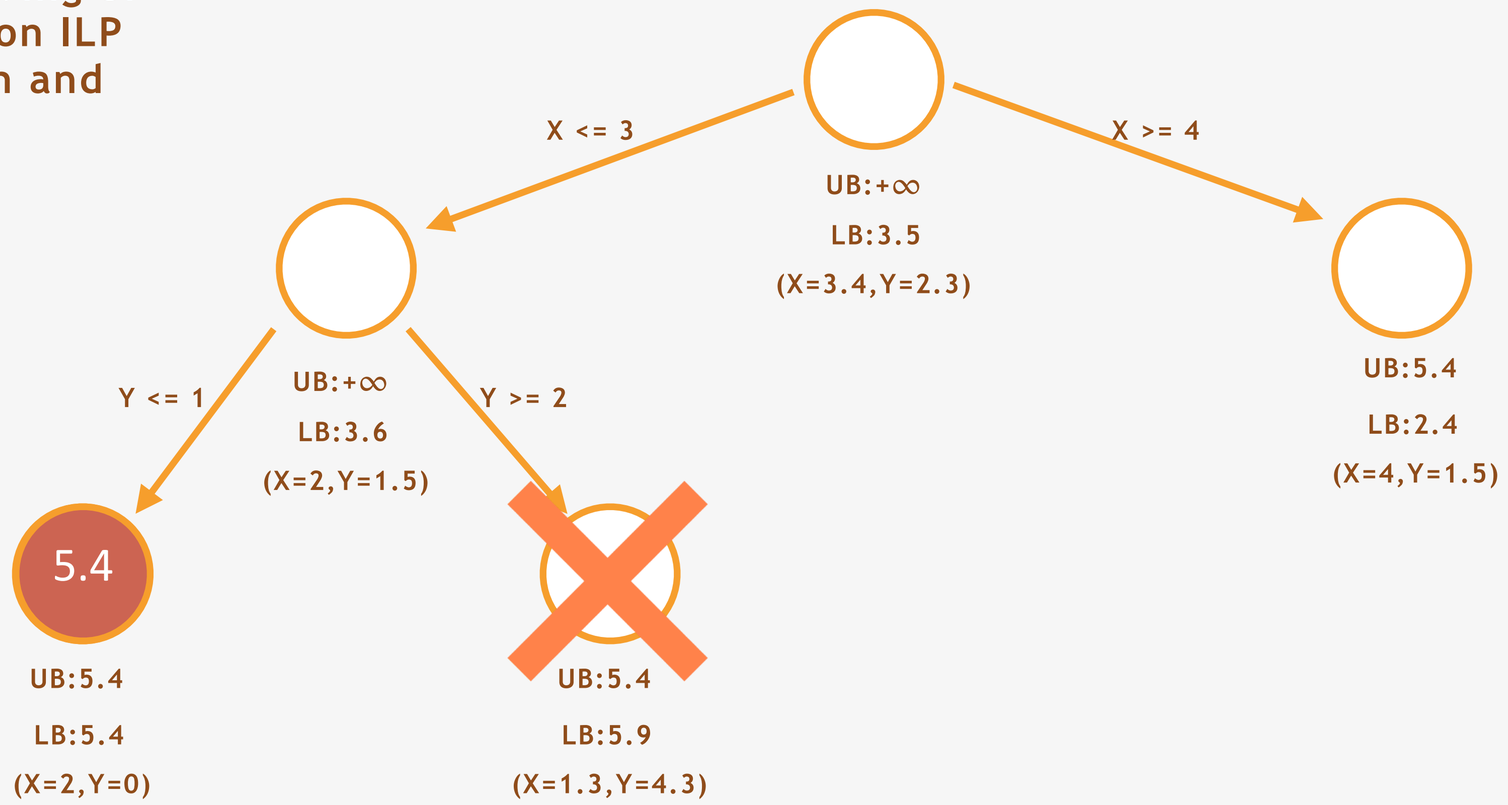

COUSIN OF THE BRANCH-AND-BOUND

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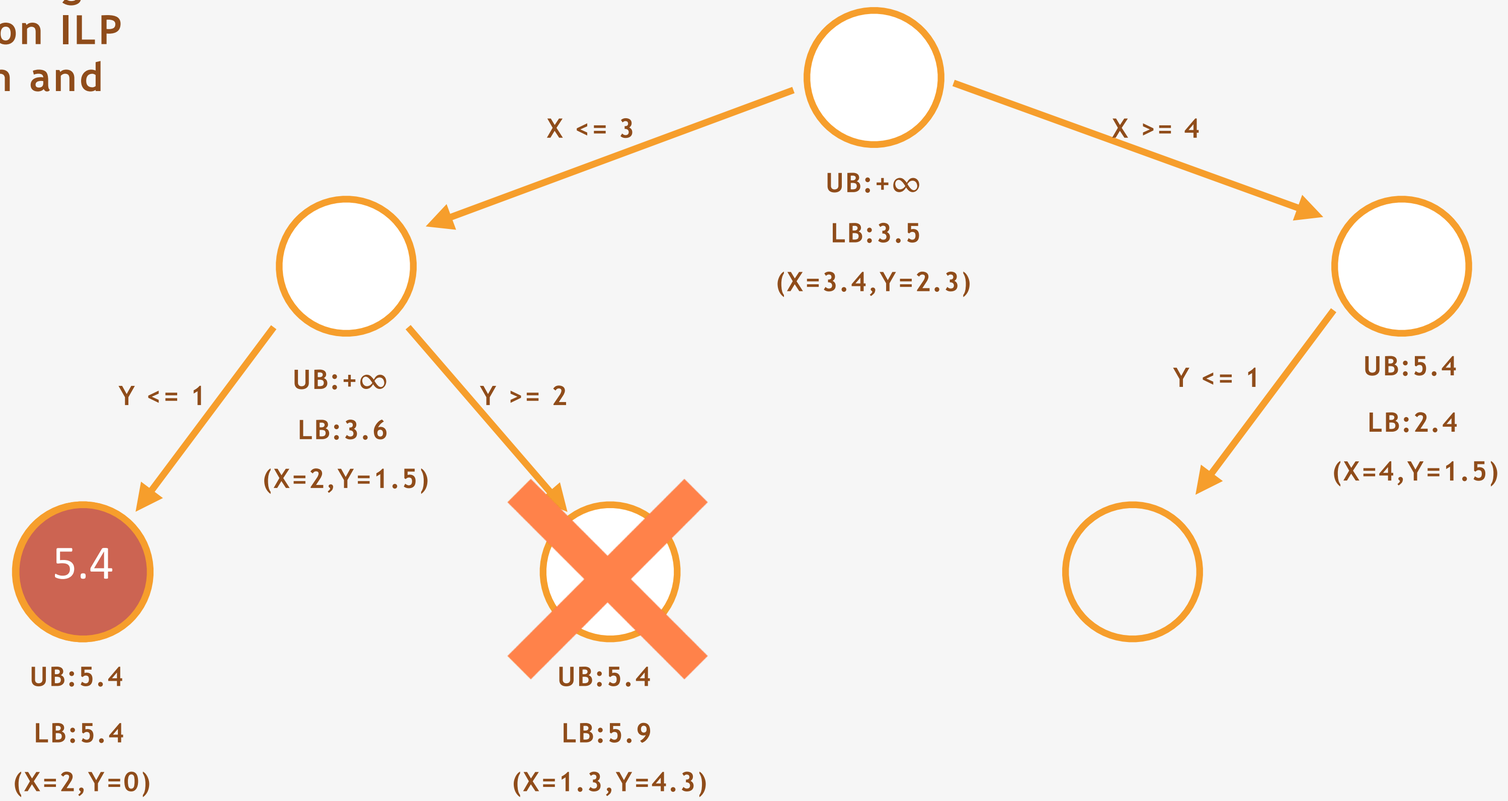

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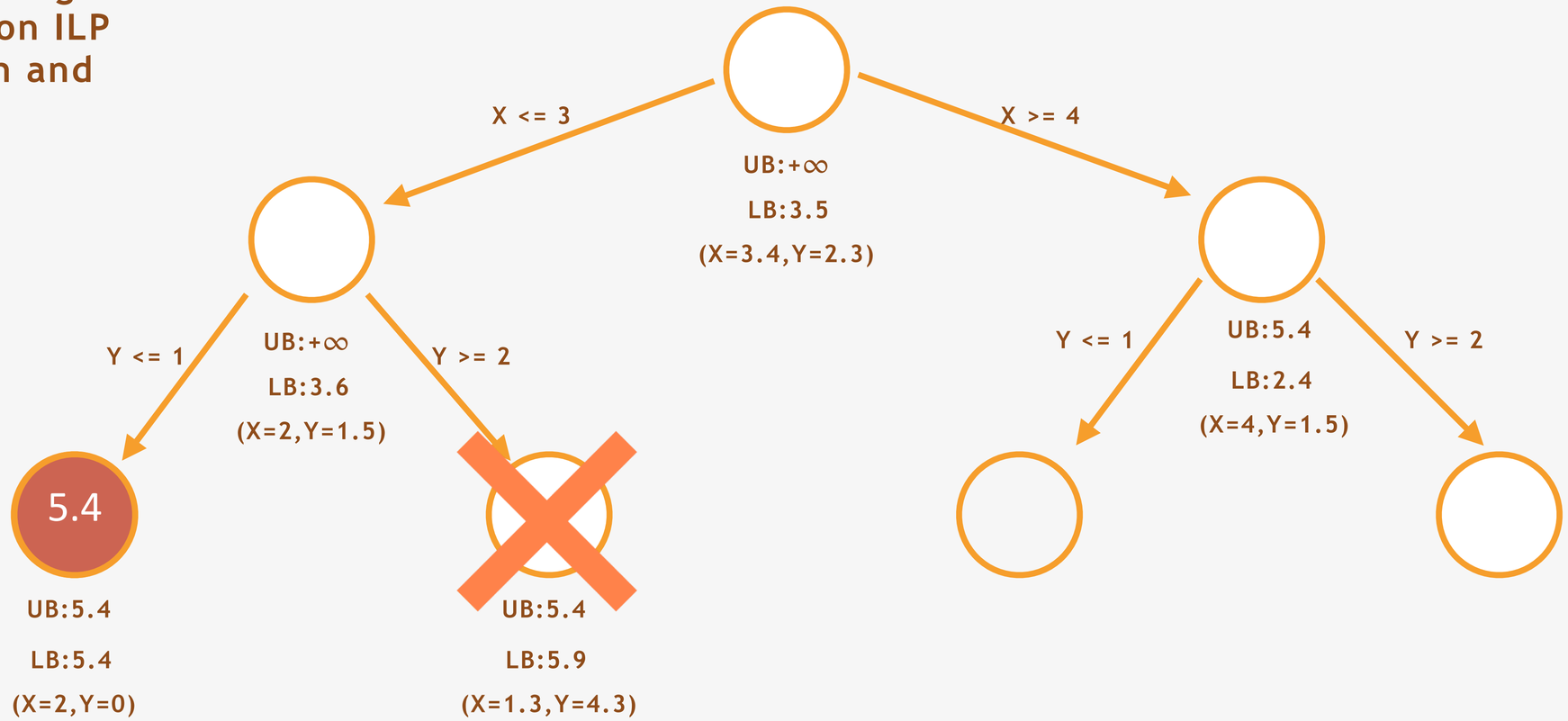

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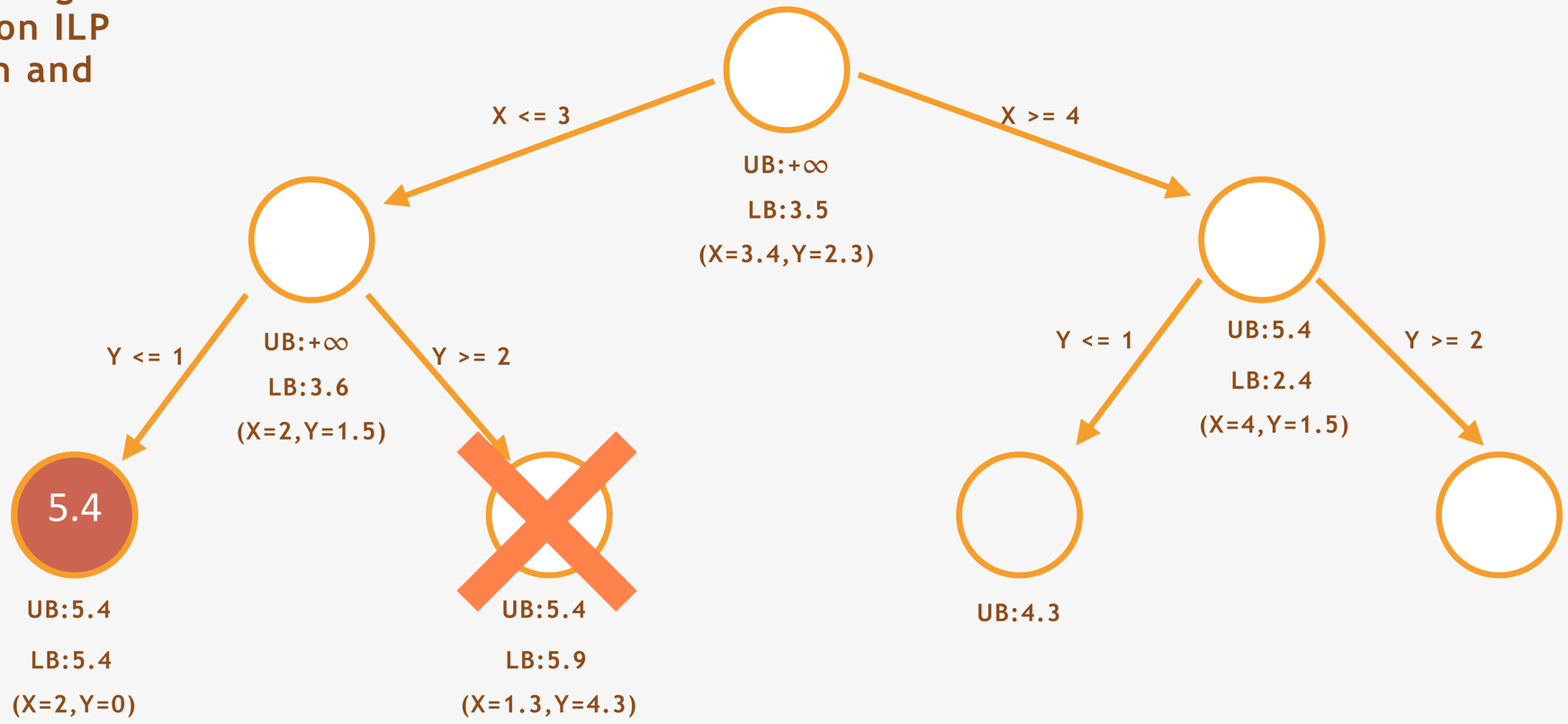

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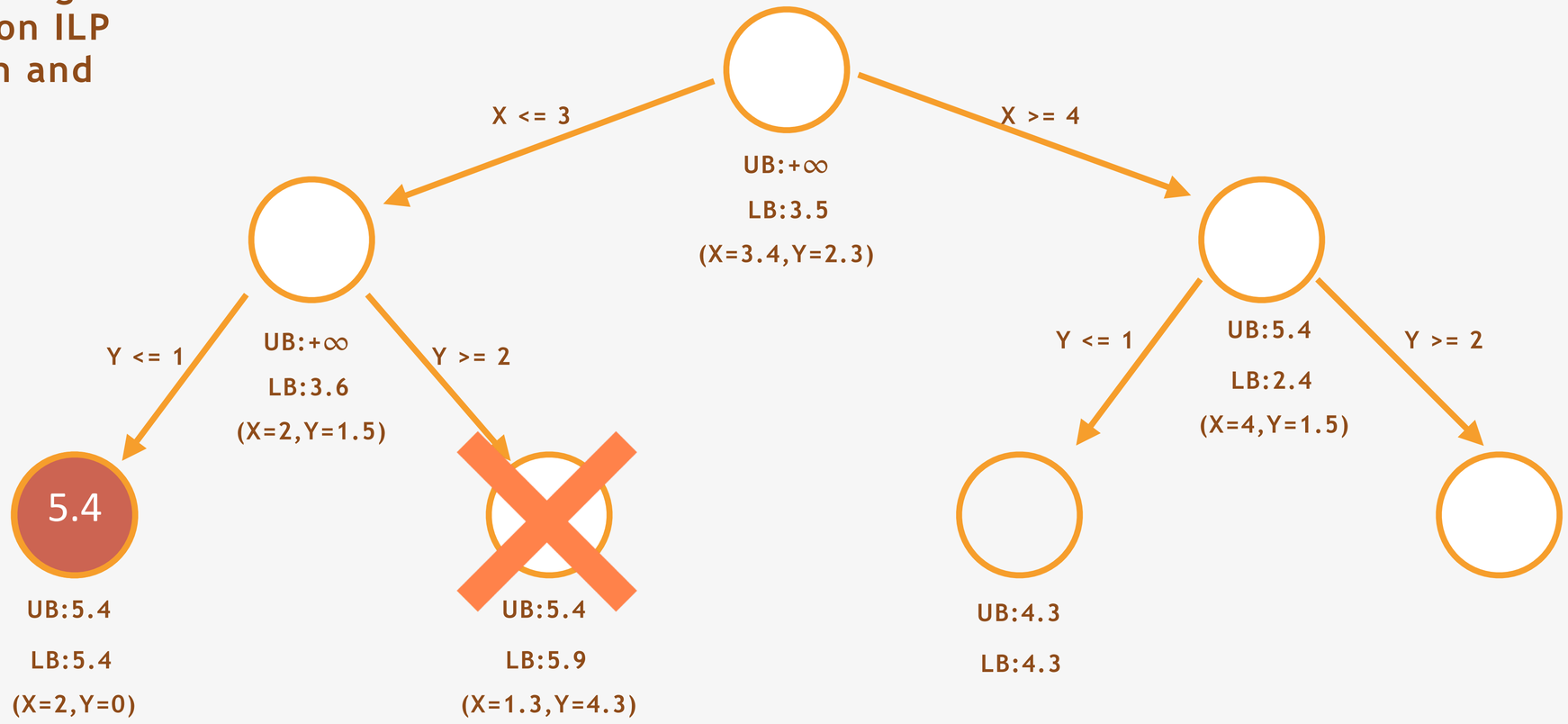

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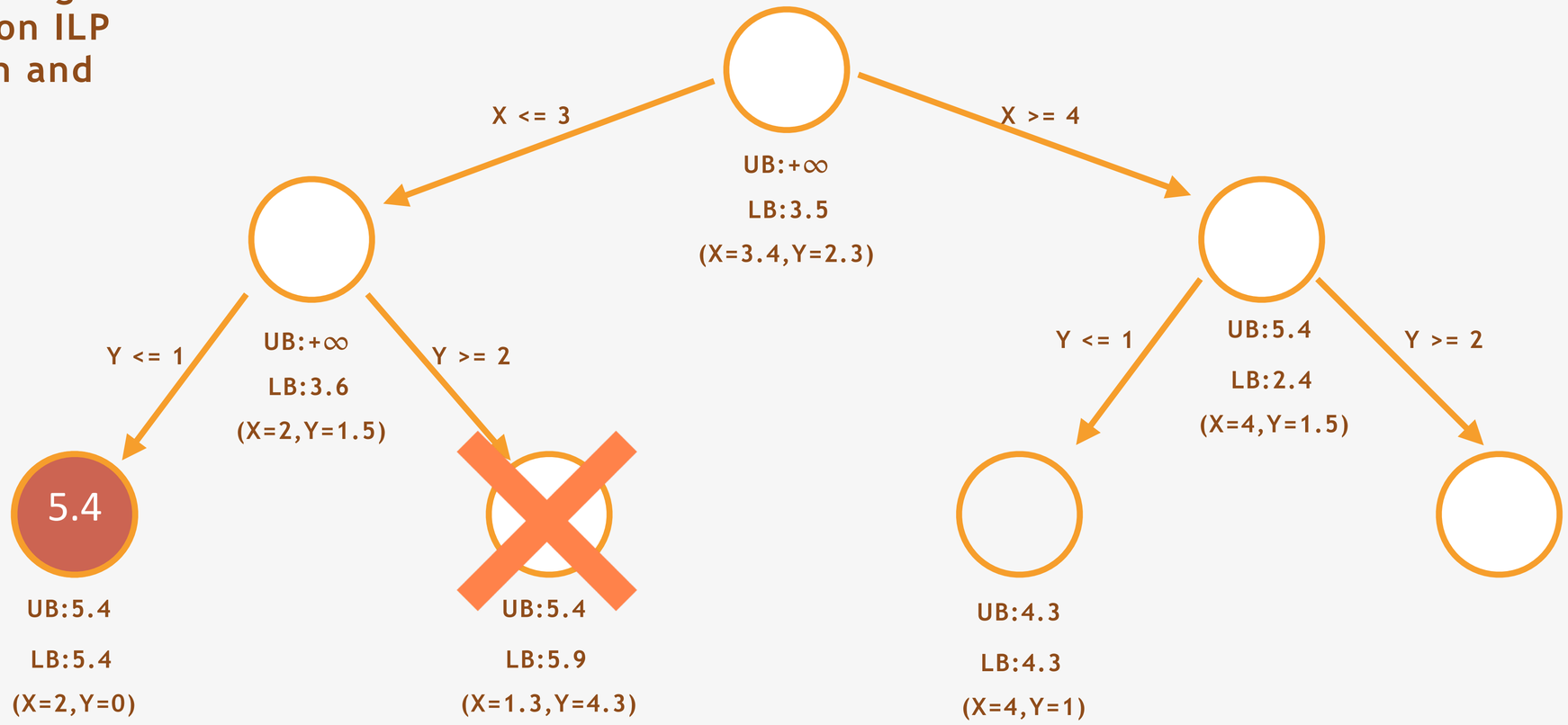

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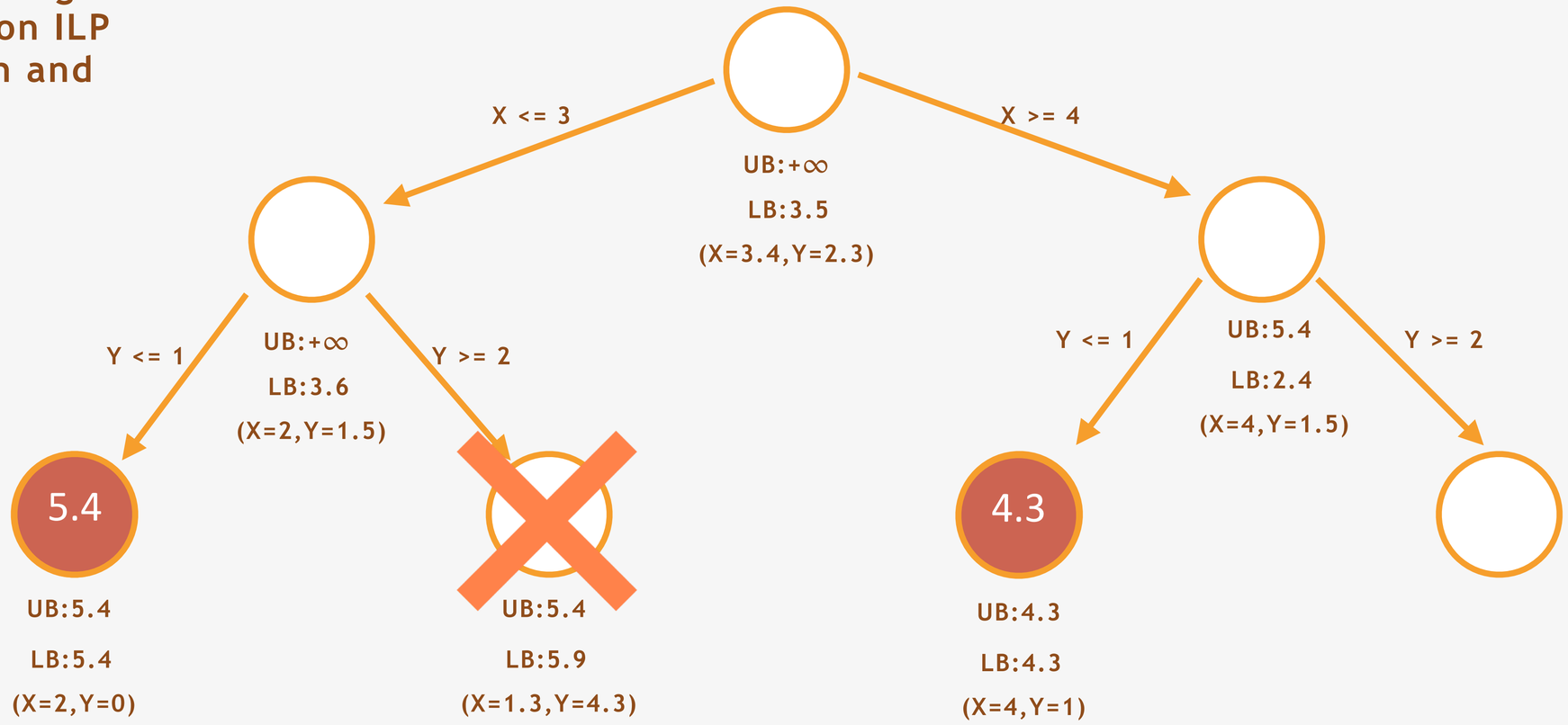

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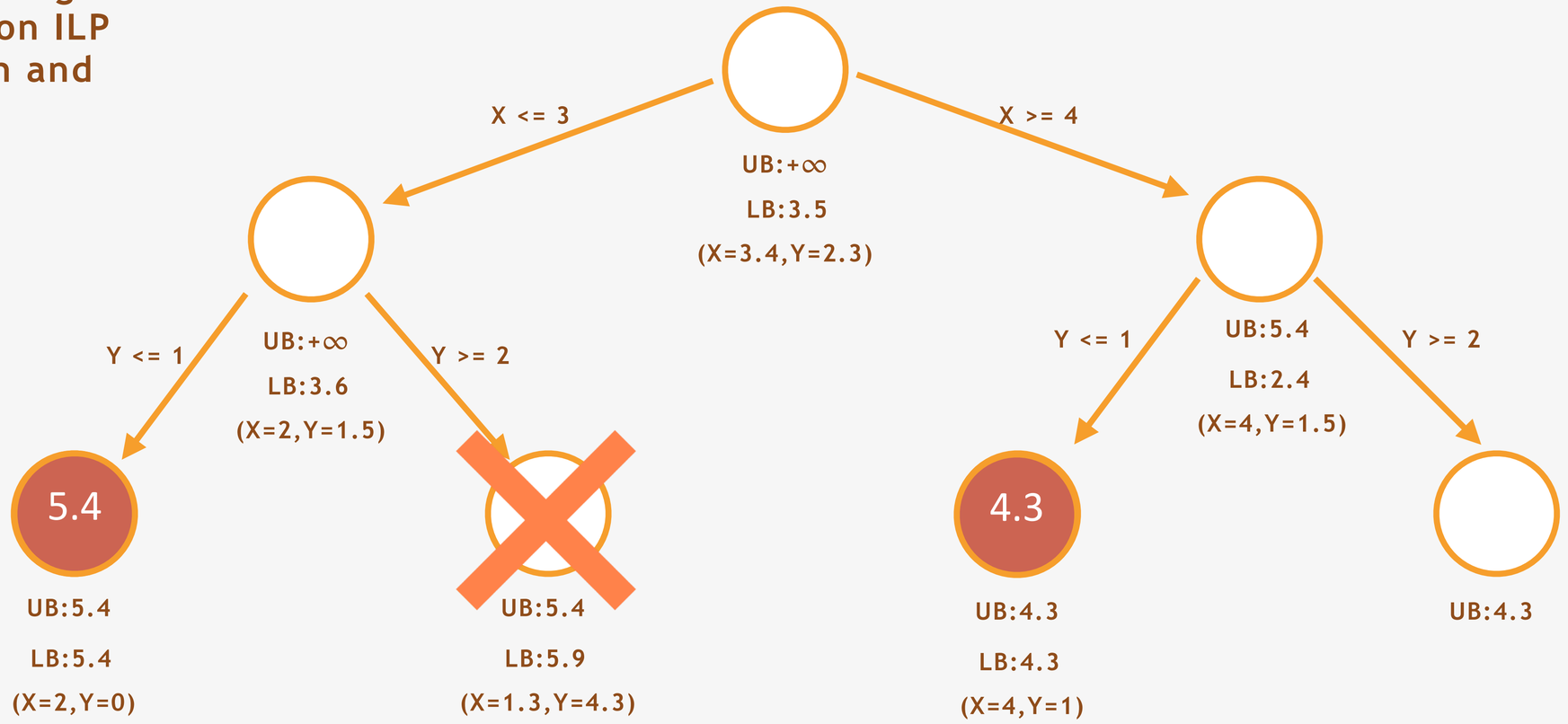

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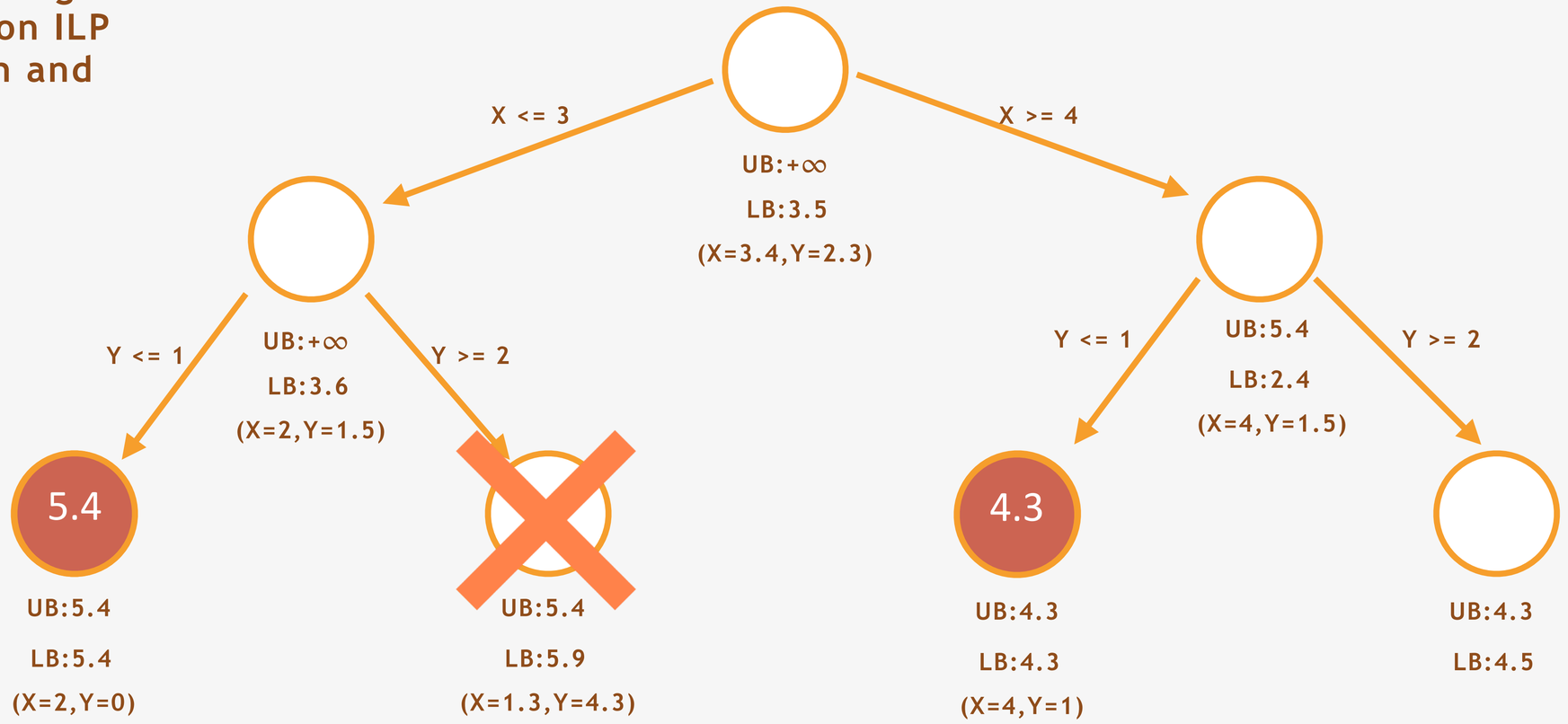

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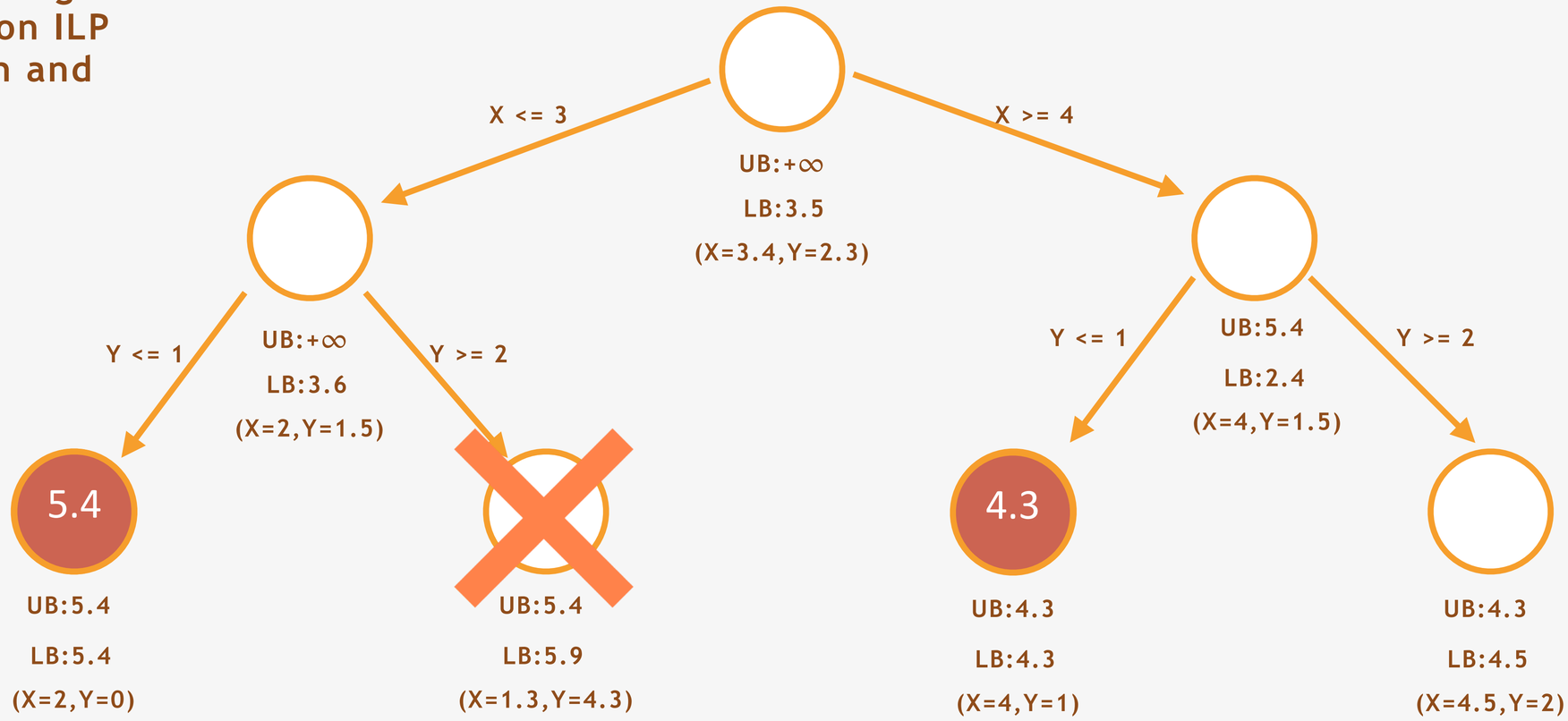

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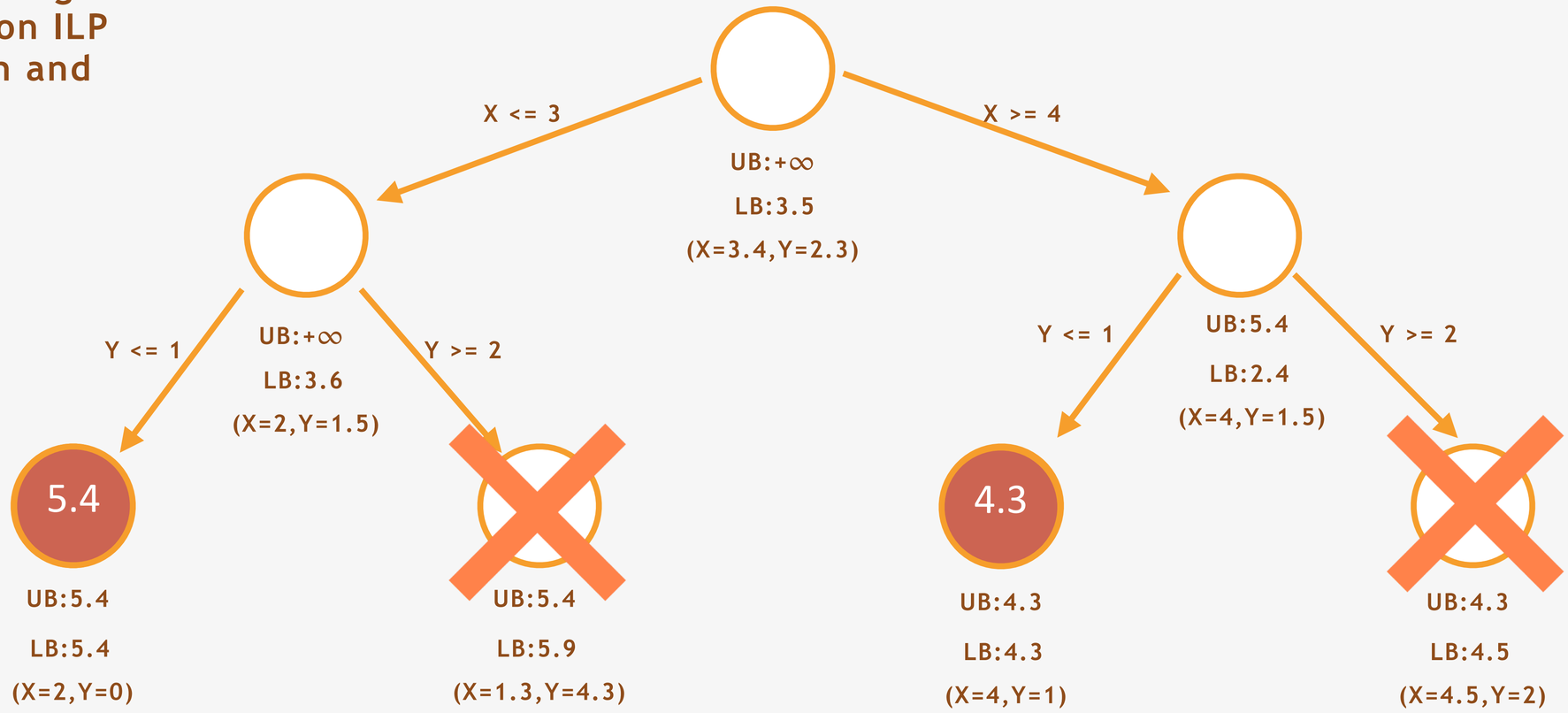

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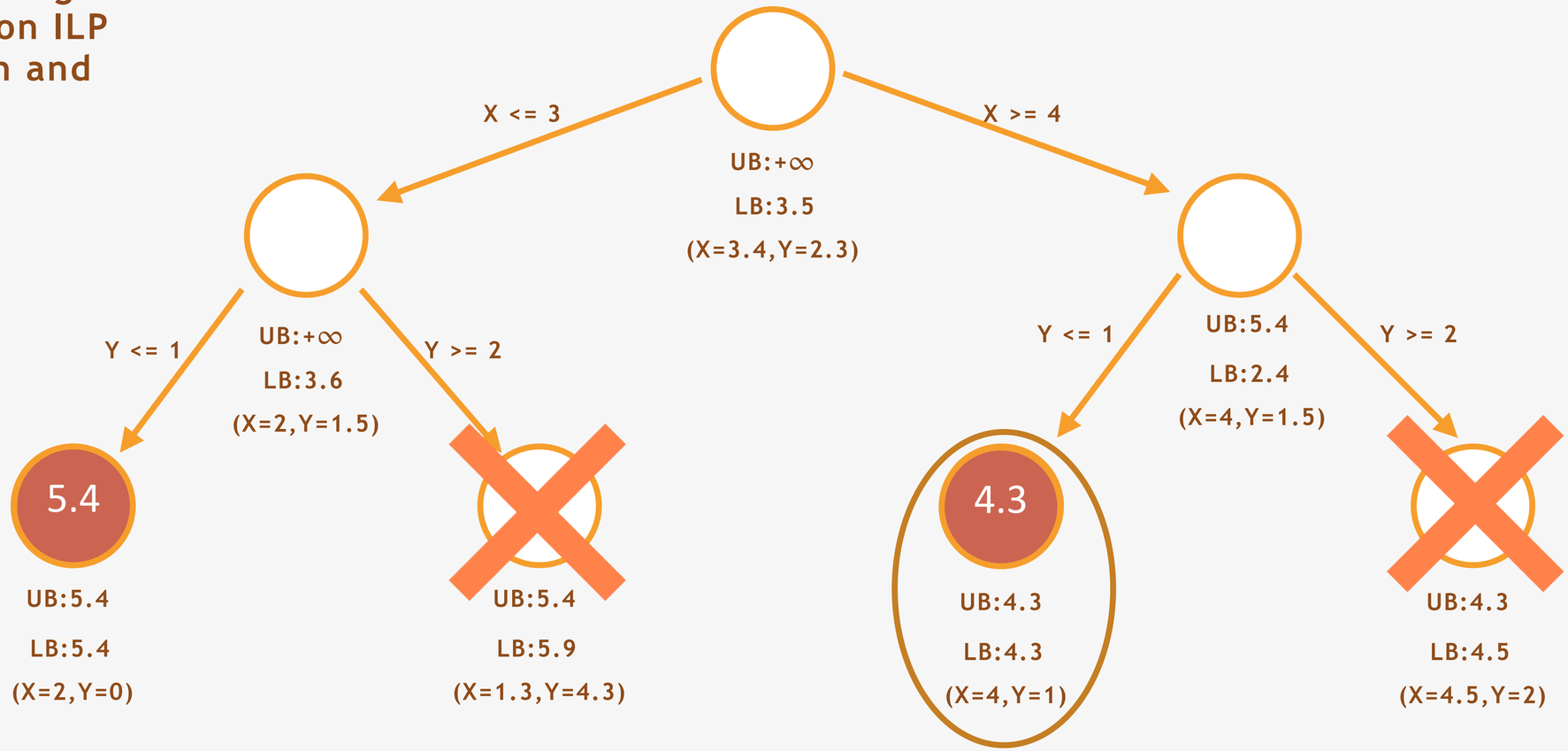

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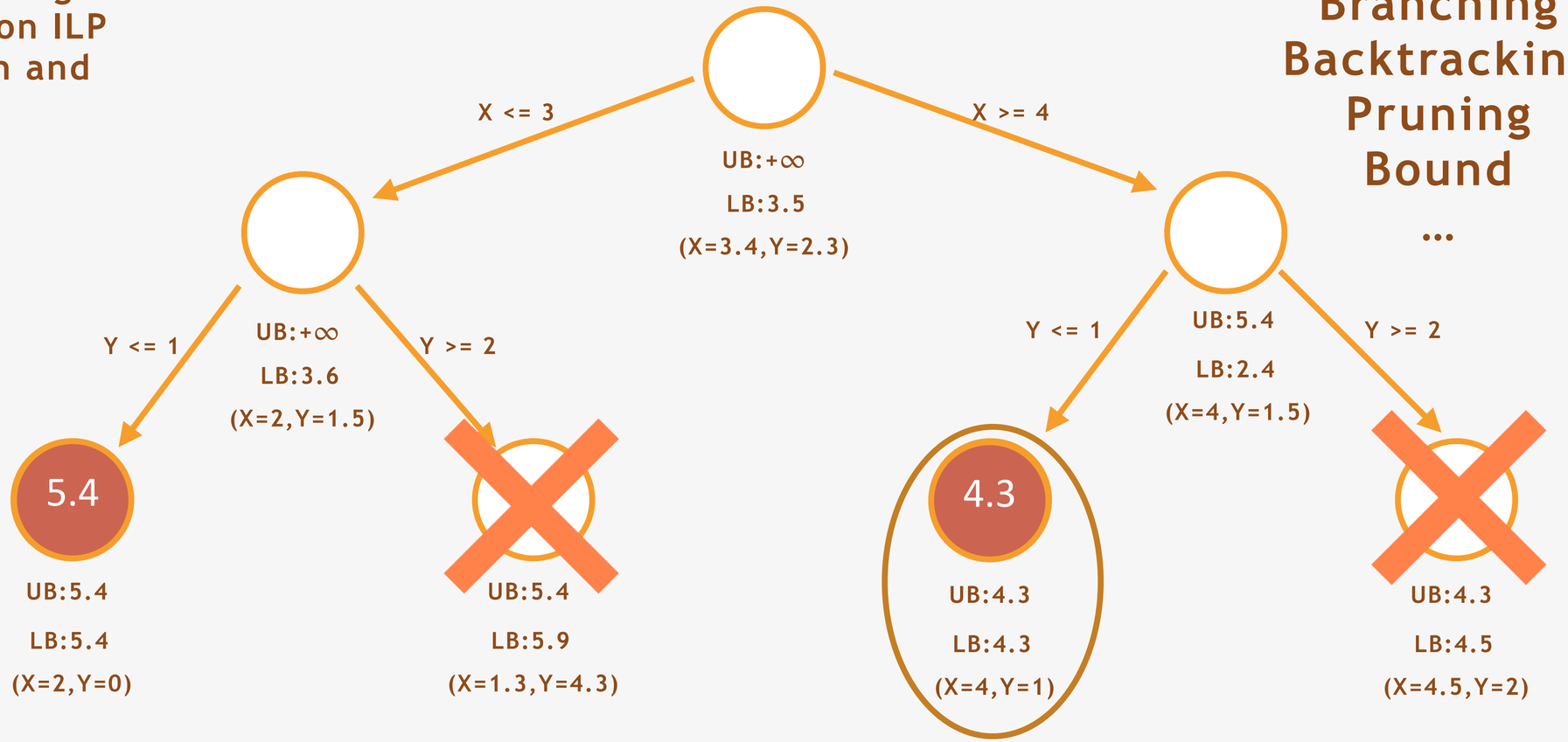

Elements Also Found in CP:

Branching
Backtracking
Pruning
Bound
...

SEARCH TREE IN MORE DETAILS

-Wednesday 9:00-10:30: Lecture

-Wednesday 11:00-12:30: Lecture

-Thursday 14:00-15:30: Lab

Advanced Solver Techniques

Hybrid solvers:

- Combination of the strengths of the various paradigms

SOME ADVANCED SOLVER TECHNIQUES

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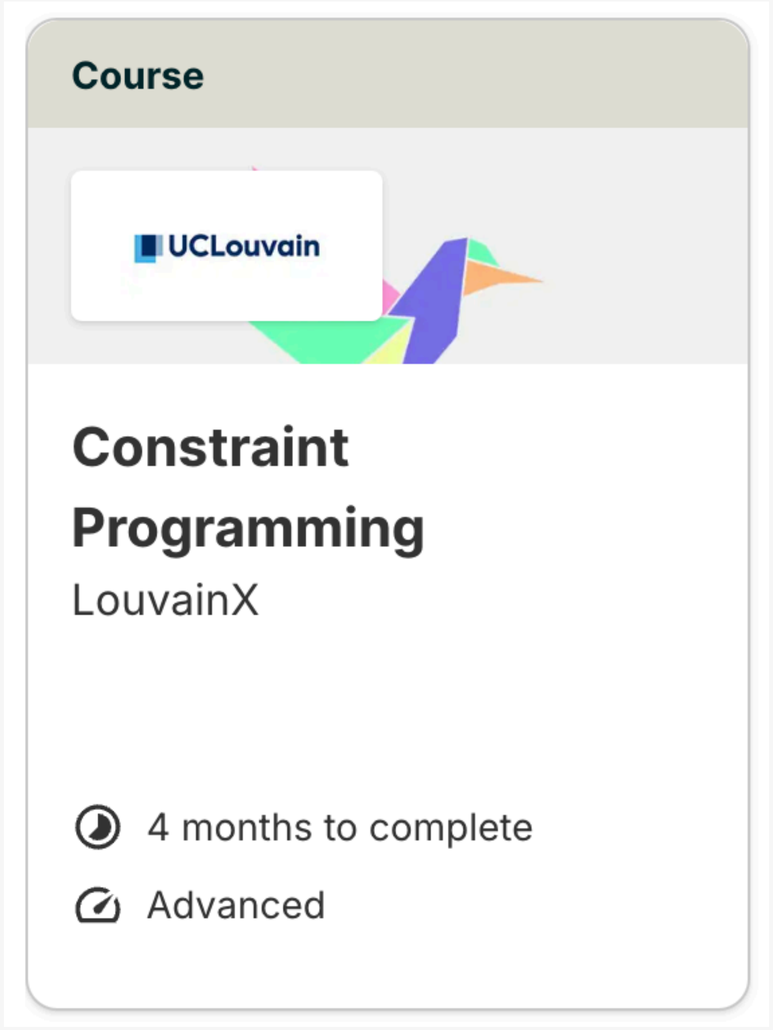
To Go Further…

ONLINE MOOC ON EDX

www.edx.org



Modelling problems, implementing global constraints, design searches,…

ONLINE MOOC ON COURSERA

Ouidah, Benin
25-29th August 2025

www.coursera.org

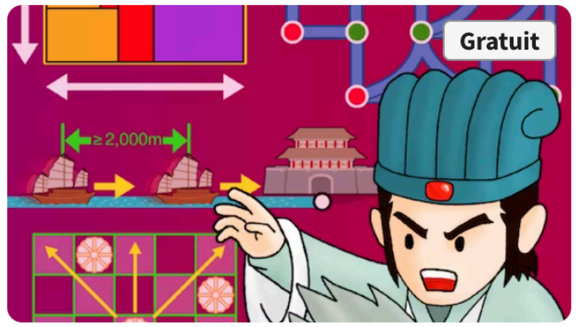
The University of Melbourne

Basic Modeling for Discrete Optimization

Compétences que vous acquerrez:
Programmation informatique

★ **4.8** (432 avis)

Intermédiaire · Cours · 1 à 4 semaines

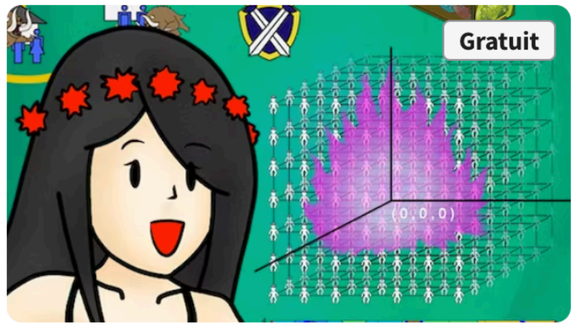


The University of Melbourne

Advanced Modeling for Discrete Optimization

★ **4.9** (131 avis)

Intermédiaire · Cours · 1 à 3 mois



The University of Melbourne

Solving Algorithms for Discrete Optimization

★ **4.8** (40 avis)

Intermédiaire · Cours · 1 à 4 semaines

Modelling of Combinatorial Optimisation Problems With MiniZinc

Solving Technologies for Combinatorial Optimisation

ACP YOUTUBE CHANNEL

Ouidah, Benin
25-29th August 2025

https://www.youtube.com/@associationforconstraintpr9021

Association for Constraint Programming

@associationforconstraintpr9021 · 475 abonnés · 53 vidéos

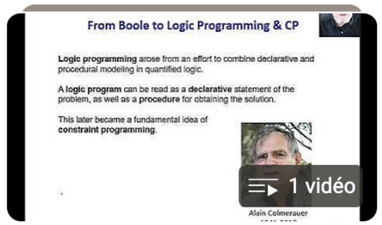
This channel features videos from the Association for Constraint Programming.

a4cp.org

S'abonner

Accueil Vidéos Playlists Communauté Chaînes

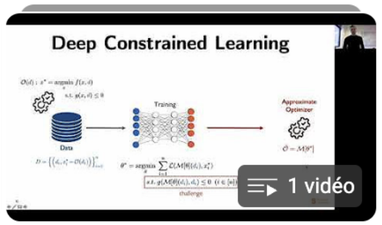
ACP Awards



Research Excellence Awards

Association for Constraint Programmin

Afficher la playlist complète



Early Career Researcher Awards

Association for Constraint Programmin

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Distinguished Service Awards

Association for Constraint Programmin

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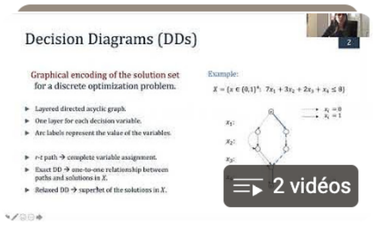

Doctoral Research Awards

Association for Constraint Programmin

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Many Open Source Solvers, Do Not Hesitate To Dive in the Documentation and the Code To Know More!

Gecode

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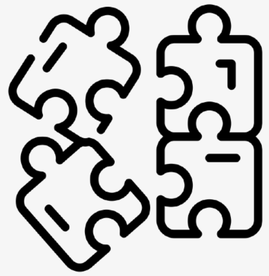
Specially Designed and Coded for Learning

Gecode

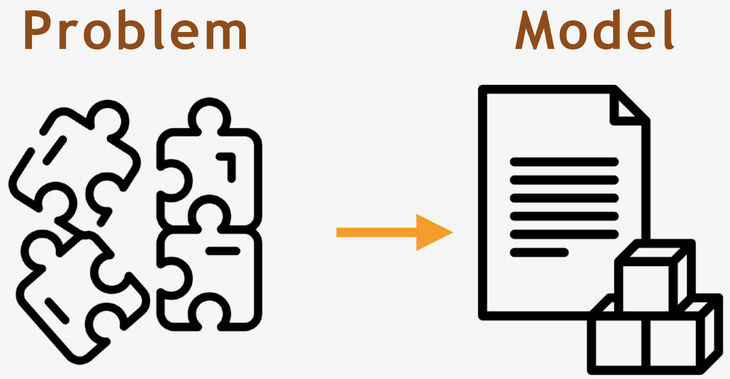
Google OR-Tools

To Conclude

Problem



TO SOLVE A PROBLEM WITH CP YOU WILL NEED...



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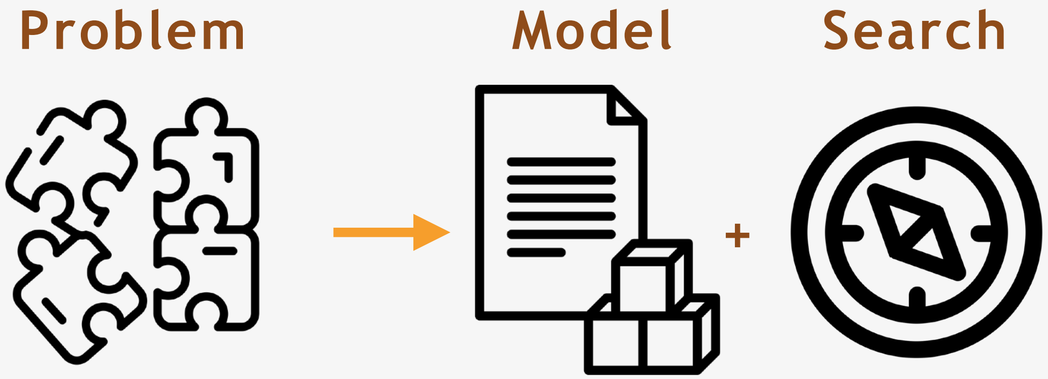

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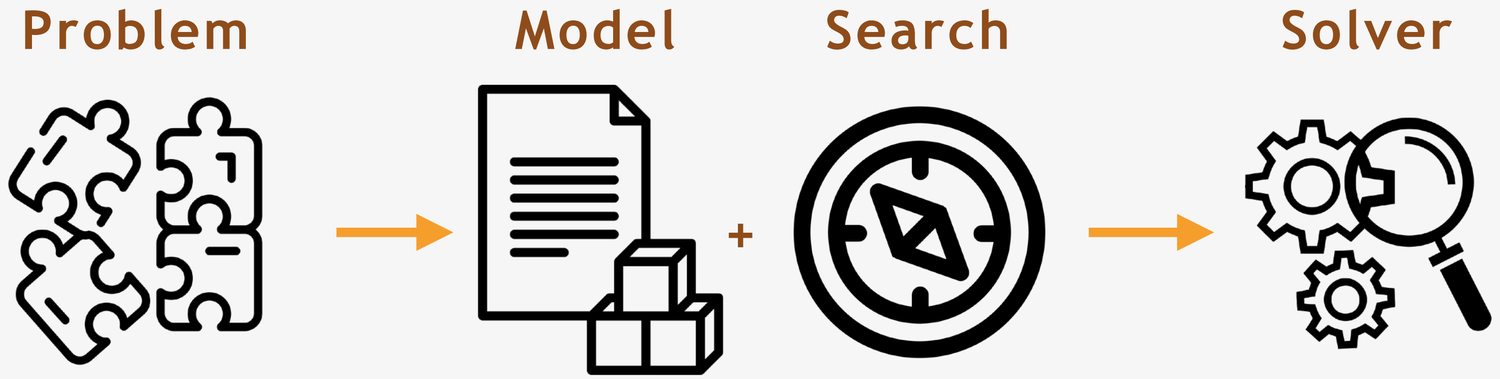

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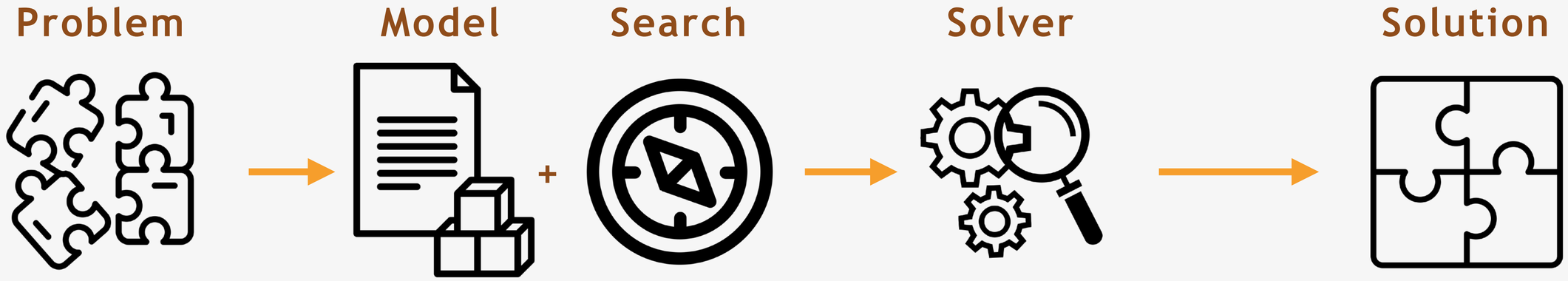

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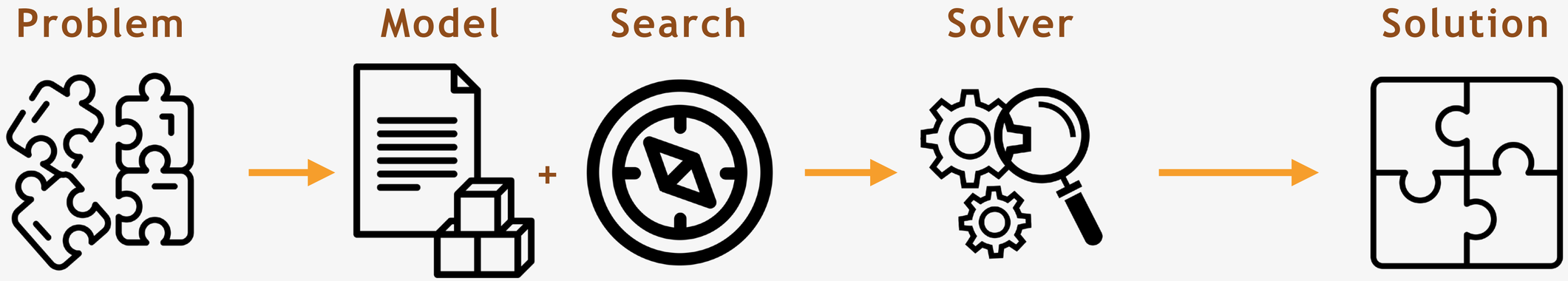

Model = declarative description of a solution to the problem, using variables and (global) constraints

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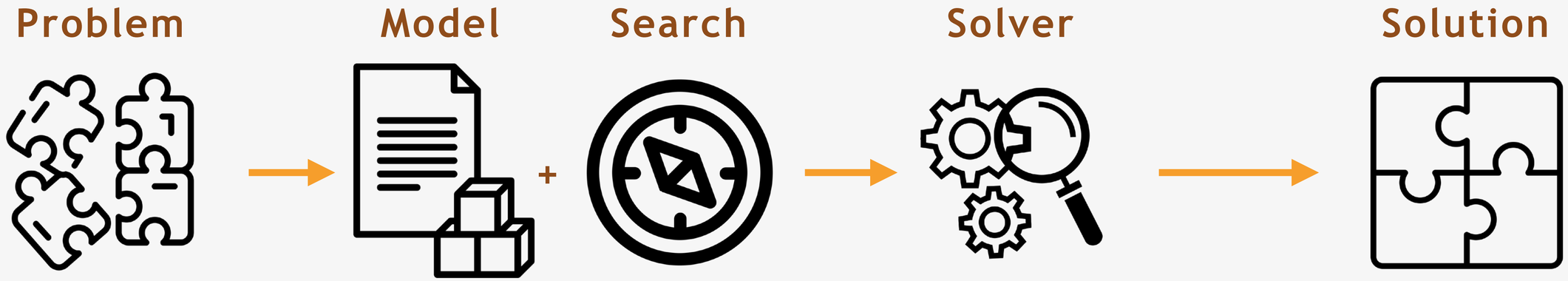

Model = declarative description of a solution to the problem, using variables and (global) constraints

Search = heuristic to guide the search tree

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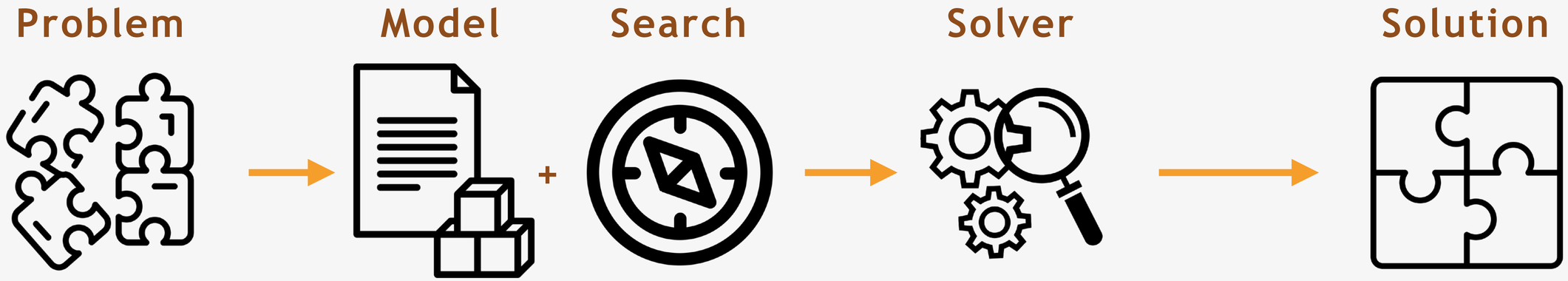

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Solver = a search tree algorithm, exploring the solution space guided by the search heuristic, which regularly call the fix-point algorithm, which organises the calls to the concerned propagators to cut the values in the state that are not valid anymore